§6.1 Linear Systems of equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \qquad\quad \vdots \qquad \ddots \qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

$$\Longleftrightarrow \quad A_{m,n}\vec{x} = \vec{b}$$

where

$$A_{m,n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

is the $m \times n$ coeficient matrix and

$$\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

is the right-hand-side vector.

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

is the vector of variables.

Recall from algebra that a linear system of equations may have a unique solution, no solutions or an infinite number of solutions.

Linear systems with no solutions are called Inconsistent. Systems with solutions are Consistent. Systems with more than one solution are called Dependent.

Matrices and Vectors

Example (i)

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \\ -3 & 4 \end{bmatrix}$$

A is 3×2 (3 rows, 2 collumns).

$a_{11} = 2$, $a_{12} = -1$, $a_{21} = 3$, $a_{22} = 5$, $a_{31} = -3$, $a_{32} = 4$.

Matric multiplication

If A is $m\times n$ and B is $n\times p$, then $C = AB$ is given by $C = [c_{ij}]$ where

$$c_{ij} = \sum_{k=1}^{n} a_{in} b_{nj} .$$

Example (ii)

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \\ -3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -2 & 1 & 5 \\ 2 & -1 & 4 & -6 \end{bmatrix}$$

then

$$AB = \begin{bmatrix} (2)(3)+(-1)(2) & (2)(-2)+(-1)(-1) & \cdots \\ (3)(3)+(5)(2) & (3)(-2)+(5)(-1) & \\ (-3)(3)+(4)(2) & (-3)(-2)+(4)(-1) & \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & -2 & 16 \\ 19 & -11 & 23 & -15 \\ -1 & 2 & 13 & -39 \end{bmatrix}$$

Note BA does not make sense, b/c B is 2×4 and A is 3×2, so the "inner" dimensions do not match.

Solving Systems of Linear Equations

Substitution: solve for, say, x_n in equation one and substitute it in to the other equations. Then repeat on the $m-1$ equations in $n-1$ variables.

Ellimination: Perform row opperations to "reduce" the coeficient matrix to the Identity.

Matrix inversion: $A\vec{x}=\vec{b} \Leftrightarrow \vec{x}=A^{-1}\vec{b}$. We will talk about this later.

We will program the Ellimination method.

Example (iii)

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 5 \\ x_1 - x_2 + x_3 &= 2 \\ 3x_1 + x_2 - 2x_3 &= -1 \end{aligned}$$

Do in class.

Look at algorithm 6.1