§6.2 Pivoting Strategies

Example (i)

$$\left[\begin{array}{cccc|c} 0 & 1 & 3 & 5 & 7 \\ 0 & -1 & 2 & -3 & 4 \\ 2 & -2 & 2 & 3 & 6 \\ 4 & 3 & 2 & 1 & 1 \end{array}\right]$$

Clearly we can not use a_{11} as the "pivot". That would produce a division by zero error.

Strategy (1): Swap row 1 with the first non-zero "pivot" row.

$$\begin{array}{cccc|cl} 2 & -2 & 2 & 3 & 6 & \leftarrow r3 \\ 0 & -1 & 2 & -3 & 4 & \\ 0 & 1 & 3 & 5 & 7 & \leftarrow r1 \\ 4 & 3 & 2 & 1 & 1 & \end{array}$$

then proceed as before ...

The problem with this is that error propagation is exacerbated when the devisor is small. The first non zero pivot may still be small (relative to the other elements).

<u>Strategy (2)</u>: Swap row 1 with the largest pivot row. (partial pivoting) Algorithm 6.2

$$\left[\begin{array}{cccc|c} 4 & 3 & 2 & 1 & 1 \\ 0 & -1 & 2 & -3 & 4 \\ 2 & -2 & 2 & 3 & 6 \\ 0 & 1 & 3 & 5 & 7 \end{array}\right] \begin{array}{l} \leftarrow r4 \\ \\ \\ \leftarrow r1 \end{array}$$

then proceed as before.

<u>Example (ii)</u>

$$\left[\begin{array}{cccc|c} 0 & 1 & 3 & 5 & 7 \\ 0 & -1 & 2 & -3 & 4 \\ 2 & -2 & 2 & 3 & 6 \\ .04 & .03 & .02 & .01 & .01 \end{array}\right]$$

Here we have the same system as in example (i), jut row 4 has been divided by 100.

Strategy (3): Scaled Partial Pivoting.

Let $s_i = \max_{1 \le j \le n} |a_{ij}|$

If $s_i = 0$ for some i, then no unique solution.
then let p be such that

$$\frac{|a_{p1}|}{s_p} = \max_{1 \le k \le n} \frac{|a_{k1}|}{s_k}$$

and swap row 1 with row p. Alorithm 6.3

$s_1 = 5$, $s_2 = 3$, $s_3 = 3$, $s_4 = .04$

$\frac{|a_{11}|}{s_1} = 0$, $\frac{|a_{21}|}{s_2} = 0$, $\frac{|a_{31}|}{s_3} = \frac{2}{3}$, $\underline{\underline{\frac{|a_{41}|}{s_4} = 1}}$

So swap row 1 with row 4
and proceed as before.

Stategy (4): Full pivoting.

Just like Scaled Partial Pivoting, only swap rows and columns.

How do we swap rows or columns?

It is time consuming to swap elements in memory. Thus we only swap indexes.

In the previous example if the matrix is in A, then we generate a vector, v, such that $v(i) = i$ to start. When we wish to "swap" rows 1 and 4 we do the following:

```
tmp = v(1);
v(1) = v(4);
v(4) = tmp;
```

then $A(v,:)$ is the new "swaped" matrix.

Look at MatLab.