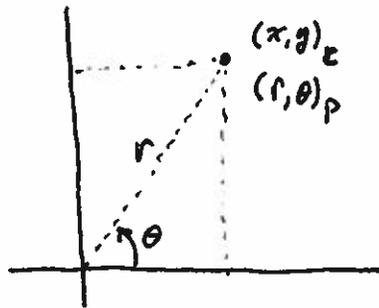


§10.3 Polar Coordinates



the point (x, y) in cartesian coordinates is (r, θ) in polar coordinates where

$$x = r \cos(\theta)$$

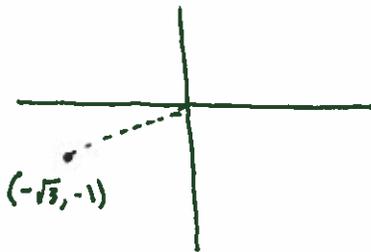
$$y = r \sin(\theta)$$

$$r^2 = x^2 + y^2$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \cot^{-1}\left(\frac{x}{y}\right)$$

These are the "change in coordinate" formulas.

Example (i) convert $(-\sqrt{3}, -1)_c$ to polar



$$r^2 = (-\sqrt{3})^2 + (-1)^2 = 4$$

so $r = 2$ (we usually have $r \geq 0$)

$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) \text{ and } \theta \text{ in } Q_{III}$$

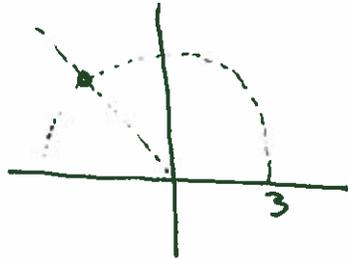
$$\text{so } \theta = \frac{7\pi}{6}$$

$$\text{thus } (-\sqrt{3}, -1)_c = \left(2, \frac{7\pi}{6}\right)_p$$

Note that the representation in polar coordinates for a point is not unique!

$$\left(2, \frac{7\pi}{6}\right)_p = \left(2, -\frac{5\pi}{6}\right)_p = \left(-2, \frac{\pi}{6}\right)_p$$

Example (ii) Convert $(3, \frac{3\pi}{4})_p$ to rectangular (cartesian) coordinates.



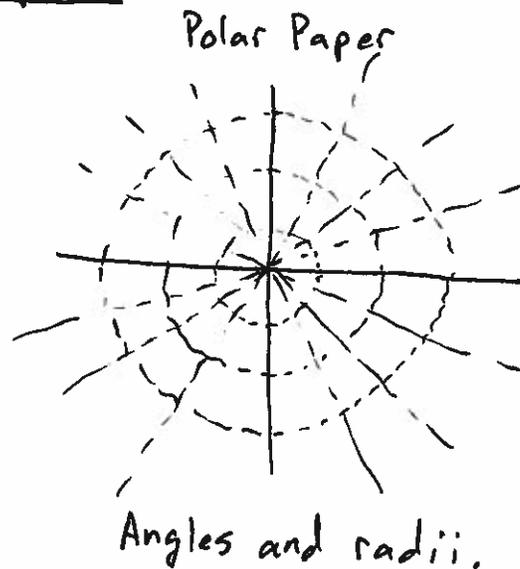
$$x = 3 \cos\left(\frac{3\pi}{4}\right) = 3\left(-\frac{\sqrt{2}}{2}\right)$$

$$y = 3 \sin\left(\frac{3\pi}{4}\right) = 3\left(\frac{\sqrt{2}}{2}\right)$$

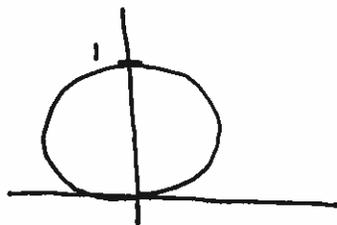
$$\text{So } (3, \frac{3\pi}{4})_p = \left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)_c$$

Note: representation of points in the cartesian coordinate system is unique!

Polar Graphs



$$r = \sin(\theta) :$$



circle

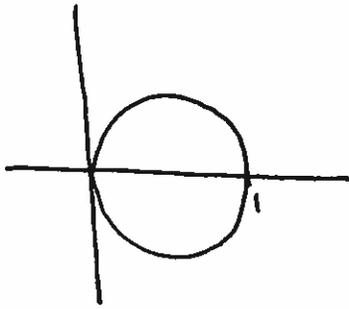
$$r^2 = r \sin(\theta)$$

$$x^2 + y^2 = y$$

$$x + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$$

$$r = \cos(\theta):$$

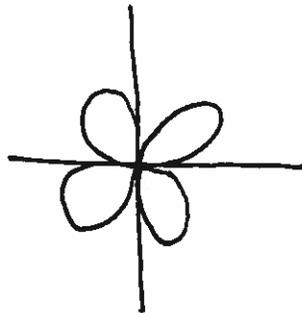
$$0 \leq \theta \leq \pi$$



$$(x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$$

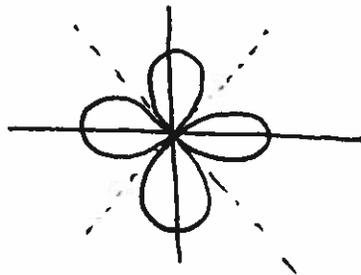
$$r = \sin(2\theta):$$

$$0 \leq \theta \leq 2\pi$$



Cartesian is hard

$$r = \cos(2\theta):$$



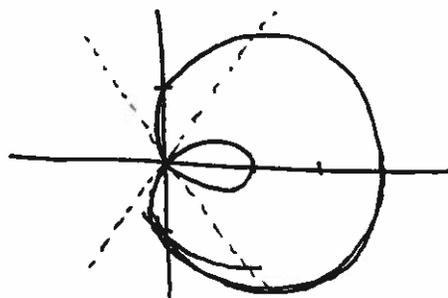
$$r = \sin(n\theta), r = \cos(n\theta) \text{ for } n=3,4,5,\dots$$

Look at graphs in Mathematica or calculator

$$r = 1 + 2\cos(\theta):$$

$$0 \leq \theta \leq 2\pi$$

Tangent at the poles
 $r = 0$, but $r \neq 0$



$$r = 0 \text{ when}$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

Note that $r = a = \text{constant}$ is a circle $\Leftrightarrow x^2 + y^2 = a^2$. $\theta = \alpha = \text{constant}$ is a line through the origin $\Leftrightarrow y = \tan(\alpha)x$.

We can also convert polar to parametric:

$$r = f(\theta) \Leftrightarrow \begin{cases} x = f(\theta) \cos(\theta) \\ y = f(\theta) \sin(\theta) \end{cases}$$

Calculator: Radian and Polar mode.

Mathematica: PolarPlot (The default is radians)