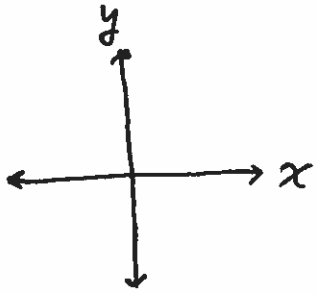
12.1 Three Dimensional Coordinate Systems

1-D  the number line

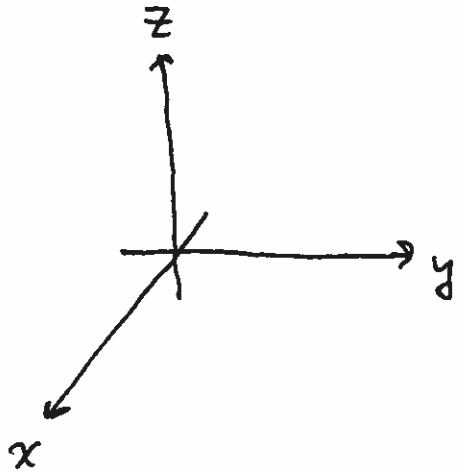
2-D 

the plane

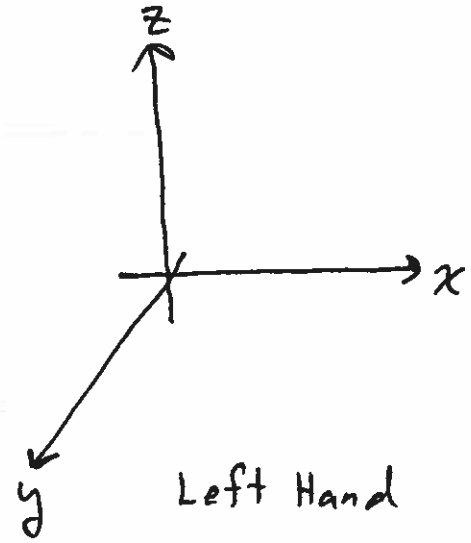
$y = f(x)$ functions

$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ parametric

$r = f(\theta)$ polar

3-D 

Right Hand

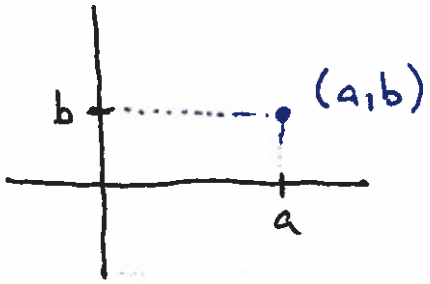


Left Hand

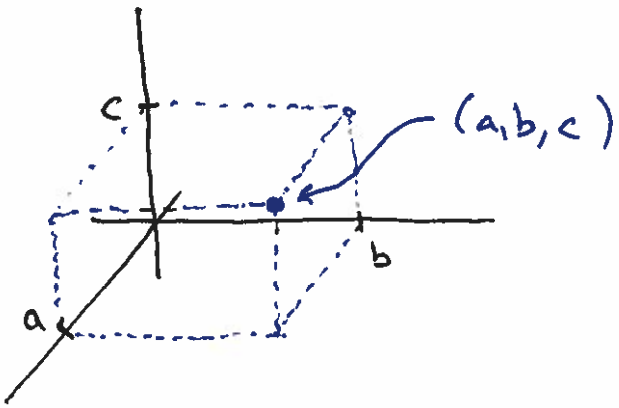
two choices: put your fingers in the direction of the x-axis and "curle" them towards the y-axis, then your thumb points in the direction of the z-axis. We will use the Right Handed Coordinate system.

Graphing points:

2-D

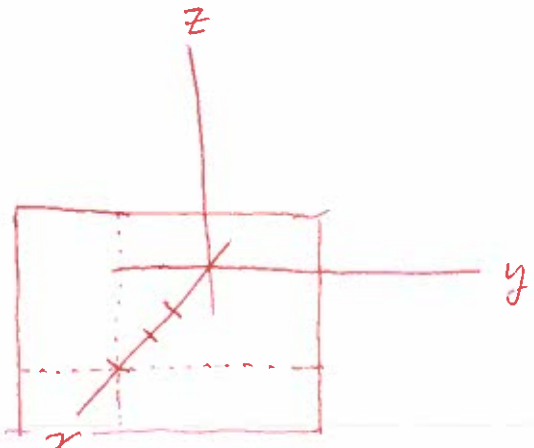


3-D



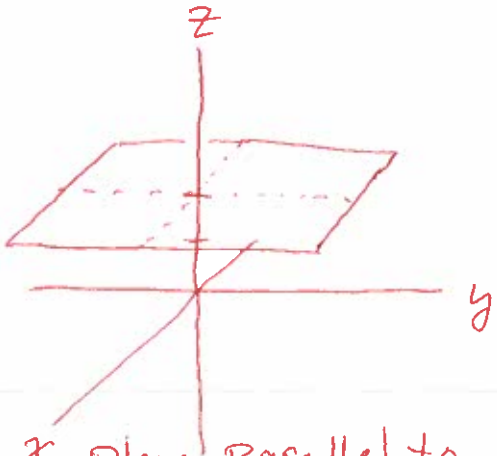
Example

$x = 3$



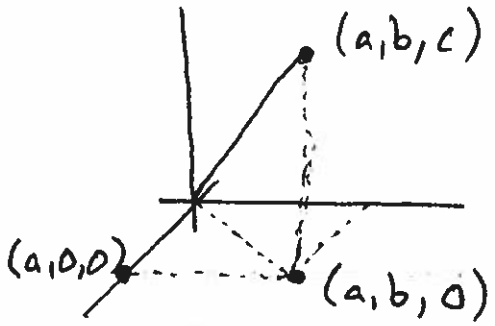
plane parallel to yz-plane

$z = 2$



z plane pasallel to xy-plane

Distance from the Origin



Use the right triangles: one in the xy-plane and the other perpendicular to the xy-plane

$$D = \sqrt{a^2+b^2+c^2}$$

Distance Between Two Points

"shift" the origin to one of the points

Let $P_1 : (x_1, y_1, z_1)$ and $P_2 : (x_2, y_2, z_2)$

then

$$|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Equation of a Sphere:

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

is centered at (h,k,l) with radius r.

Example

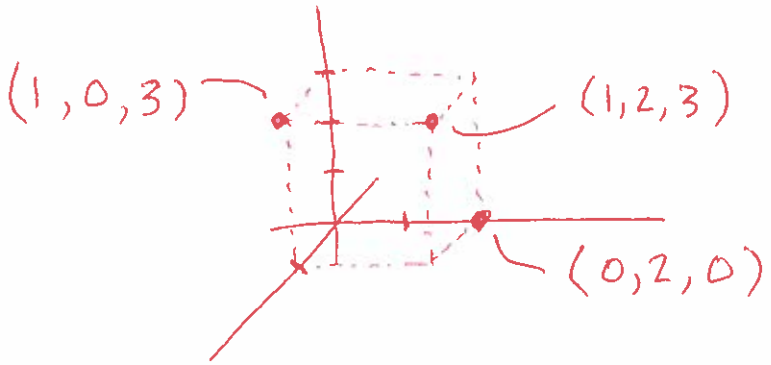
Find the distance from the point $(1,2,3)$ to:

a) y-axis

b) the xz-plane

c) the point $(2,-1,5)$

First, draw a picture



a) the "closet" point on the y-axis is $(0,2,0)$, so the distance is

$$\sqrt{(1-0)^2+(2-2)^2+(3-0)^2} = \sqrt{10}$$

b) the "closet" point in the xz-plane is $(1,0,3)$, so the distance is

$$\sqrt{(1-1)^2+(2-0)^2+(3-3)^2} = 2$$

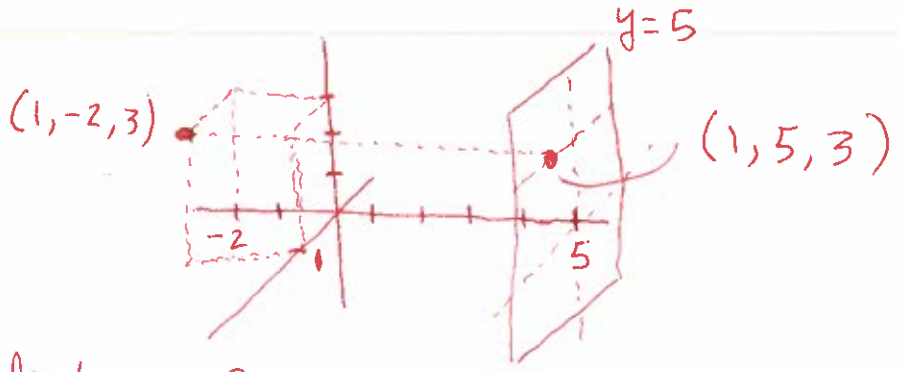
c) Use distance formula

$$\sqrt{(1-2)^2+(2-(-1))^2+(3-5)^2} = \sqrt{14}$$

Example

Find the equation of the largest sphere centered at $(1,-2,3)$ that does not cross the plane $y=5$ (it can be tangent).

Draw a picture:



The distance from $(1,-2,3)$ to $y=5$ is 7.
This is the radius of the sphere.
So

$$(x-1)^2+(y+2)^2+(z-3)^2=49$$