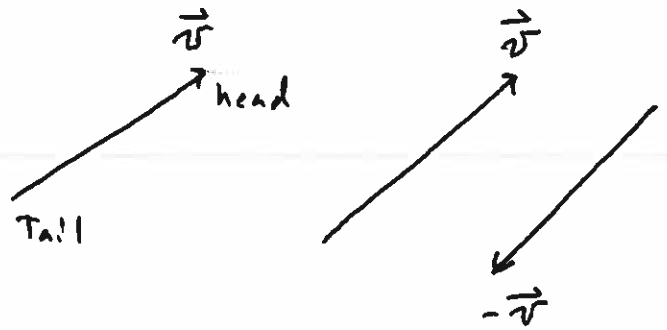
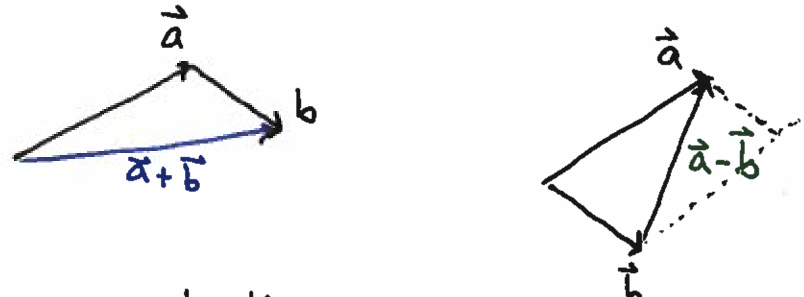
§12.2 Vectors

<u>Def</u> A vector is an object with direction and magnitude.

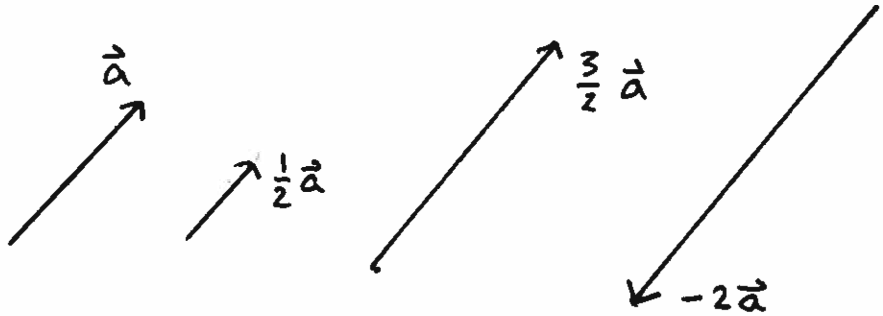


These are suppose to be parallel and have the same length. Placement on the page does not matter. The minus sign "flips" the direction.

<u>Combining vectors</u> (Geometrically)

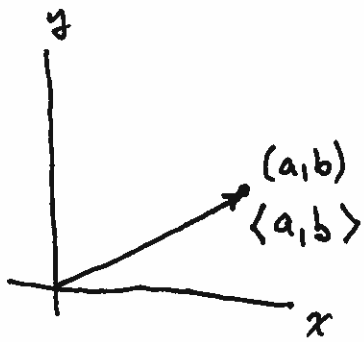


Looking at the parallelagram induced by the two vectors, one diagonal is the sum and the other is the difference of the vectors.



Multiplying by a scalar (real number) changes the magnitude of the vector, but not the direction.

<u>Components</u>:



With it's tail on the origin, the head of the vector $\langle a,b \rangle$ points to the point (a,b).

The x-component of $\langle a,b \rangle$ is a, the y-component is b.

<u>Magnitude</u>

The length or magnitude of the vector $\vec{v} = \langle a, b, c \rangle$ is the distance between the origin and the point (a,b,c). That is

$$|\vec{v}| = \sqrt{a^2 + b^2 + c^2}$$

Example

Find the magnitude of the vector with head at $P(3,7,1)$ and tail at the point $Q(1,2,3)$.

We find the components of $\overrightarrow{QP}$ by taking the difference in the coordinates (head − tail):

$$\overrightarrow{QP} = \langle 3-1, 7-2, 1-3 \rangle$$
$$= \langle 2, 5, -2 \rangle$$

Then

$$|\overrightarrow{QP}| = \sqrt{2^2+5^2+(-2)^2} = \sqrt{33}$$

Arithmatic of Vectors

Let $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$ be vectors in $\mathbb{R}^2$ (2-D) and let $c \in \mathbb{R}$ (c is in the set of real numbers). Then

$$c\vec{a} = \langle ca_1, ca_2 \rangle \quad \text{and}$$
$$\vec{a}+\vec{b} = \langle a_1+b_1, a_2+b_2 \rangle$$

That is, addition and scalar multiplication are performed component-wise.

Properties see page 819

Example

Prove property 3 on page 819 for vectors in $\mathbb{R}^3$.

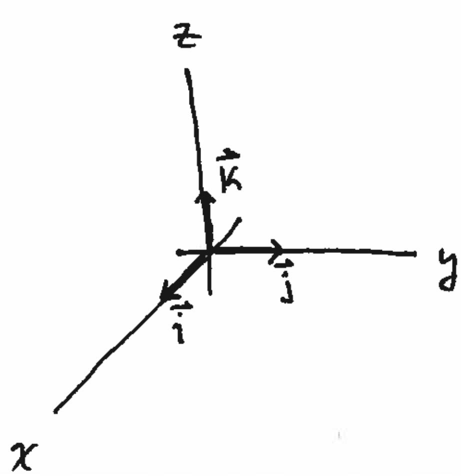
First: there is a "zero" vector $\vec{0} = \langle 0,0,0 \rangle$ in $\mathbb{R}^3$. If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ in $\mathbb{R}^3$, then

$$\vec{a} + \vec{0} = \langle a_1, a_2, a_3 \rangle + \langle 0,0,0 \rangle$$
$$= \langle a_1 + 0, a_2 + 0, a_3 + 0 \rangle$$
$$= \langle a_1, a_2, a_3 \rangle \text{ by properties of real numbers}$$
$$= \vec{a}$$

$\blacksquare$

Standard Basis Vectors

In $\mathbb{R}^3$, $\vec{i} = \langle 1,0,0 \rangle$, $\vec{j} = \langle 0,1,0 \rangle$, $\vec{k} = \langle 0,0,1 \rangle$



so $\langle a_1, a_2, a_3 \rangle =$
$a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$

Unit Vectors

$\vec{u} = \frac{1}{|\vec{a}|}\vec{a}$ is the unit vector in the direction of $\vec{a}$, provided $\vec{a} \neq \vec{0}$.

Note 1: $\vec{0}$ either does not have a direction, or has all directions, depending on the situation.

Note 2: $\vec{u} = \frac{1}{|\vec{a}|}\vec{a}$ has magnitude $\underline{1}$, provided $\vec{a} \neq \vec{0}$.

Applications

Position, velocity, acceleration and force are all vectors. The magnitude of velocity is called speed.