12.6 Cylinders and Quadric Surfaces

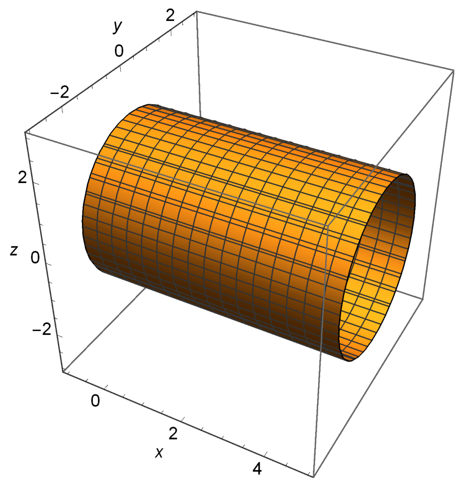
Cylinders

Definition: A **cylinder** is a surface that consists of all lines that are parallel to a given line and pass through a given curve.

Example: a right-circular cylinder $y^2 + z^2 = 4$.

In[1]:=
```
ContourPlot3D[y^2 + z^2 == 4, {x, -1, 5}, {y, -3, 3}, {z, -3, 3}, AxesLabel → Automatic]
```

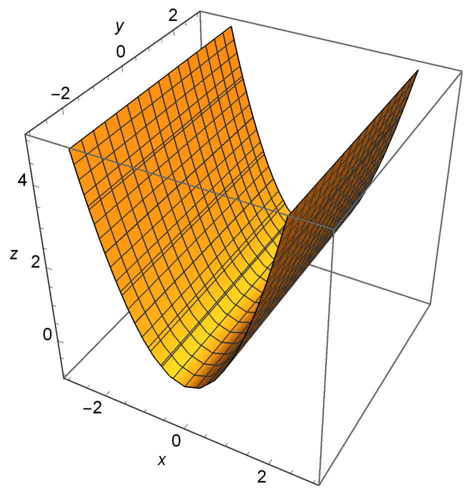
Out[1]=



Example: a parabolic cylinder $z = x^2$.

In[2]:=
```
ContourPlot3D[z == x^2, {x, -3, 3}, {y, -3, 3}, {z, -1, 5}, AxesLabel → Automatic]
```

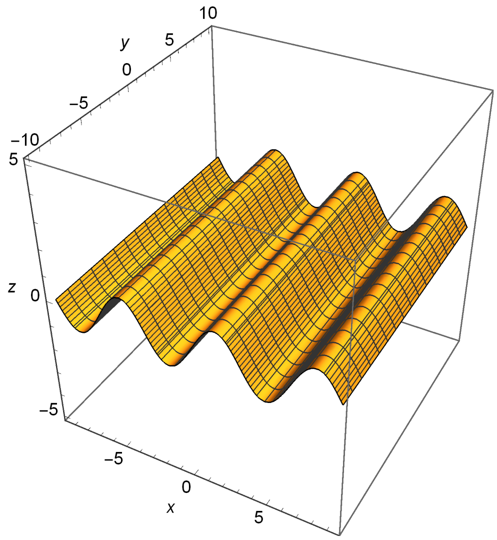
Out[2]=



Example: a sinusoidal cylinder $z = \sin(x)$.

In[3]:=
```
ContourPlot3D[z == Sin[x], {x, -3 Pi, 3 Pi}, {y, -10, 10}, {z, -5, 5}, AxesLabel → Automatic]
```

Out[3]=



Quadric Surfaces

Definition: a **Quadric Surface** is the graph of a second-degree polynomial equation in the variables *x*, *y*, and *z*.

The most general form for this type of equation is:

$$A x^2 + B y^2 + C z^2 + D x y + E y z + F x z + G x + H y + I z + J = 0.$$

However, through the use of translations, rotations and change of variables it can be reduced to one of the two forms:

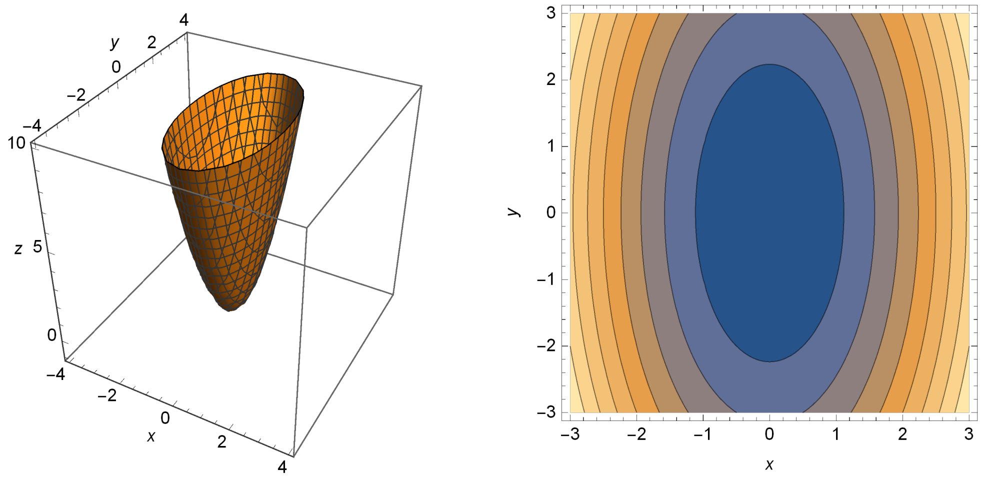
$$a x^2 + b y^2 + c z^2 + d = 0, \text{ or } z = a x^2 + b y^2.$$

See page 854 for a table of the different types of quadric surfaces. We will look at a few examples here.

Example: elliptic paraboloid

In[10]:=
```
p1 = ContourPlot3D[z == 4 x^2 + y^2, {x, -4, 4}, {y, -4, 4}, {z, -1, 10}, AxesLabel -> Automatic];
p2 = ContourPlot[4 x^2 + y^2, {x, -3, 3}, {y, -3, 3}, FrameLabel -> Automatic];
GraphicsGrid[{{p1, p2}}]
```

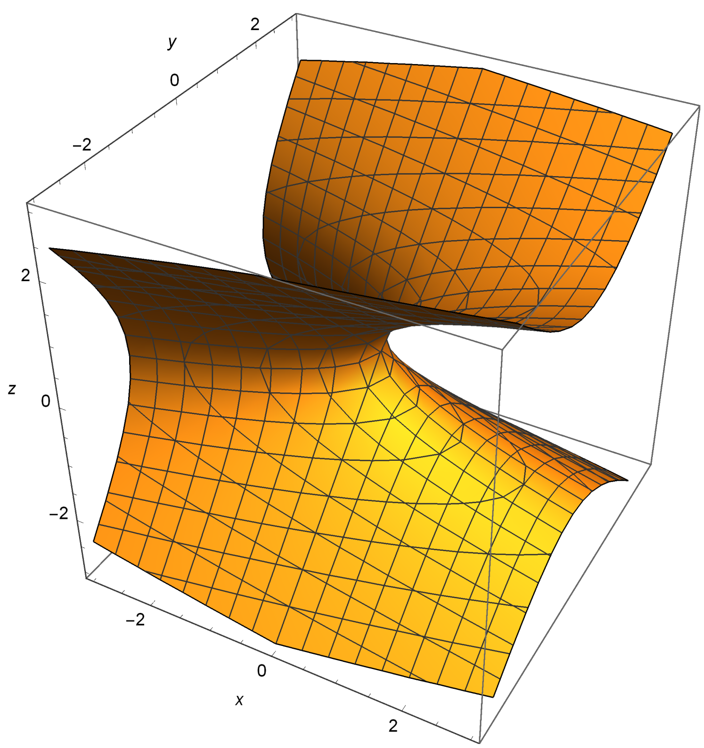
Out[12]=



Example: a hyperbolic paraboloid

In[14]:=
```
ContourPlot3D[x == z^2 - y^2, {x, -3, 3}, {y, -3, 3}, {z, -3, 3}, AxesLabel -> Automatic]
```

Out[14]=



Note that the “level curves” are parabolas when slicing for constant z and constant y and are hyperbolas for constant x.