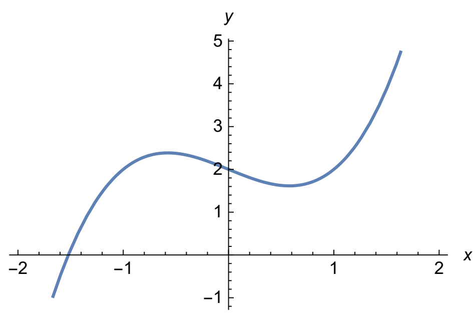
14.1 Functions of several variables

In the Past:

In calculus I and II we study functions with one independent variable and one dependent variable. We have notation like $y = f(x)$, construct tables of values and have graphs like

In[1]:=
```
Plot[x^3 - x + 2, {x, -2, 2}, AxesOrigin → {0, 0}, AxesLabel → {x, y}]
```

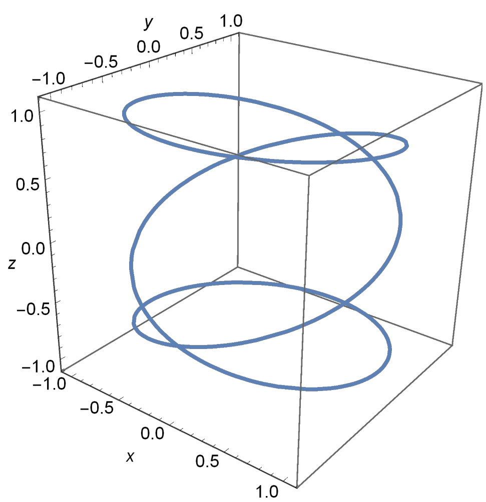
Out[1]=



In calculus III we have looked at vector functions: one independent variable and multiple dependent variables. We have notation like $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$, construct tables of values and have graphs like

In[2]:=
```
ParametricPlot3D[{Cos[3 t], Sin[3 t], Sin[t]}, {t, 0, 2 Pi}, AxesLabel → {x, y, z}]
```

Out[2]=



For these types of functions we study limits, continuity, derivatives, integrals and their applications.

Functions of Two Independent Variables

Now we look at functions with two independent variables and one dependent variable. We have a notation: $z = f(x, y)$. We can construct tables of values (see page 903 in the book). We can construct graphs by plotting points.

Example: generate a surface graph and a contour graph of the function $z = \frac{\text{Sin}[x]\,\text{Sin}[y]}{1+x^2+y^2}$.

First, function notation in Mathematica:

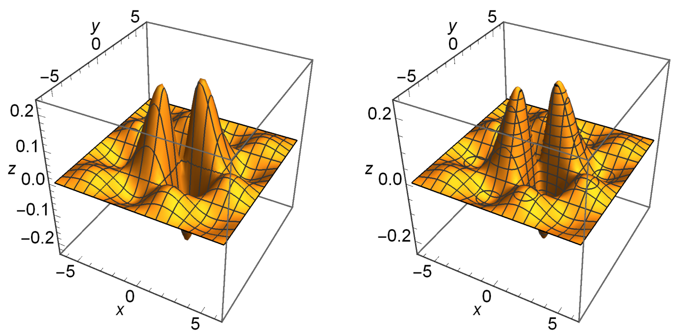
In[3]:=
```
f[x_, y_] = Sin[x] Sin[y] / (1 + x^2 + y^2)
```

Out[3]= $\frac{\mathrm{Sin}[x]\ \mathrm{Sin}[y]}{1 + x^2 + y^2}$

Next the surface graph:

In[4]:=
```
p1 = Plot3D[f[x, y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}, PlotRange → .24,
    PlotPoints → 50, AxesLabel → {x, y, z}, BoxRatios → {1,  1,  1}];
p2 = ContourPlot3D[z == f[x, y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi},
    {z, -.24, .24}, AxesLabel → {x, y, z}];
GraphicsGrid[{{p1, p2}}]
```

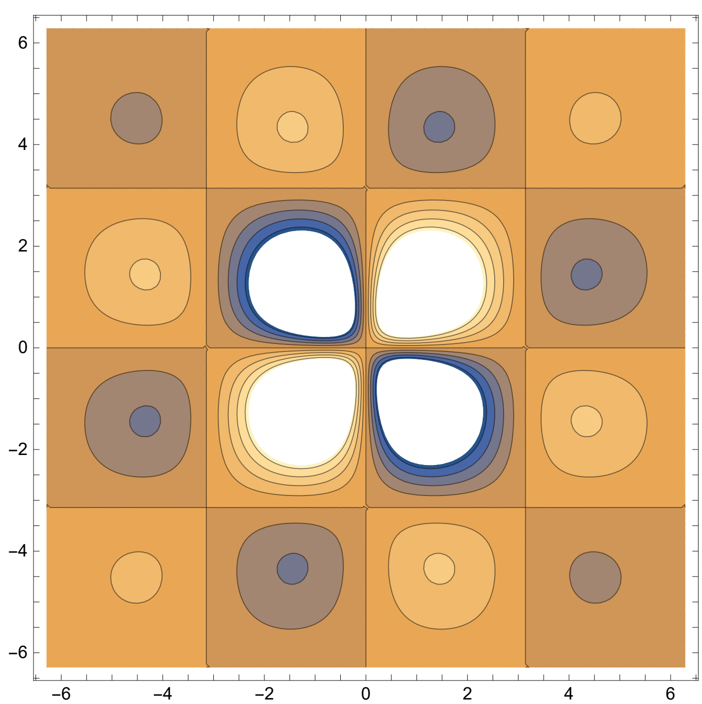
Out[6]=



Now the contour graph (plotting level curves):

In[7]:=
```
ContourPlot[f[x, y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}, PlotPoints → 50]
```

Out[7]=



Functions of Three or More Variables

Now consider the case were we have a function of three (or more) independent variables and one dependent variable. We have a notation, like: $w = f(x, y, z)$. Constructing tables of values and graphing the “surfaces” becomes problematic. However, for $w = f(x, y, z)$ we may be able to graph the level surfaces (see page 911 for an example). In any case, we can extend most of our calculus concepts from 2 to more independent variable by using our imaginations.

Final Thoughts

We will concentrate our efforts in this section to problems like 59-64 on page 915.