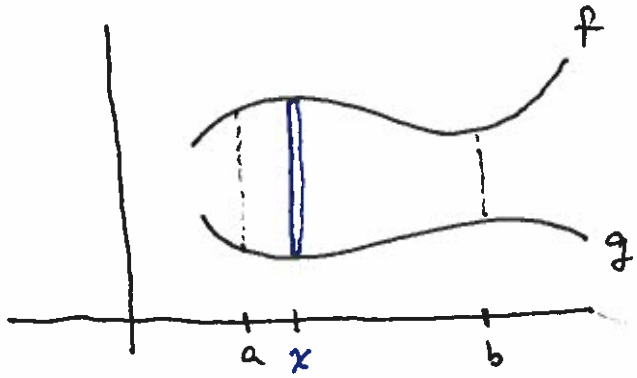
§6.1 Area between curves



A small piece of area between the curves is

$$dA = (f(x) - g(x))\,dx$$

so total are is

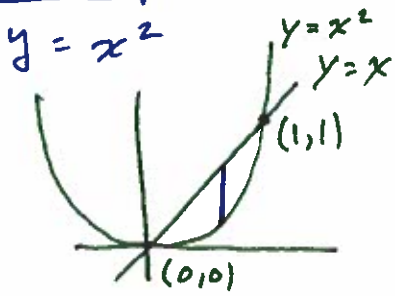
$$A = \int_a^b (f(x) - g(x))\,dx$$

<u>Definition</u>

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} (f(x_k^*) - g(x_k^*))\,\Delta x$$

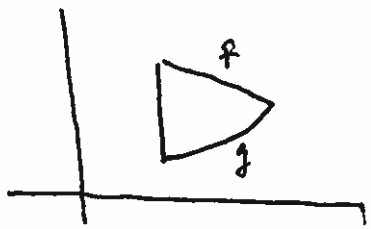
where $\Delta x = \frac{b-a}{n}$, $x_k = a + k\Delta x$ and $x_{k-1} \le x_k^* \le x_k$.

<u>Example</u> (i) Find area bounded by $y = x$ and $y = x^2$

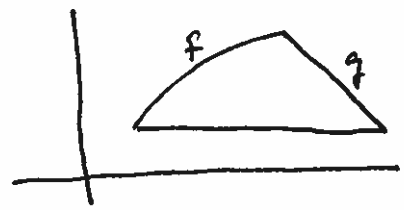


so $A = \int_0^1 (x - x^2)\,dx = \frac{1}{6}$

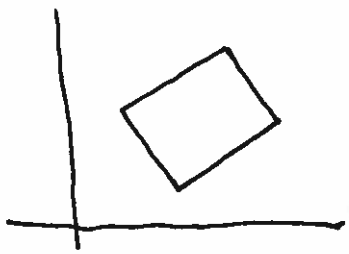
Simple Regions



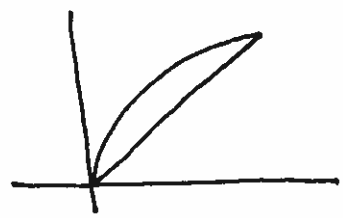
x-simple,
not y-simple



y-simple,
not x-simple

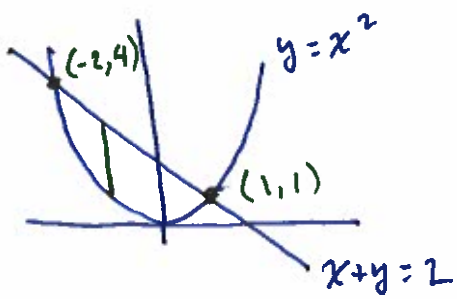


neither x-simple,
nor y-simple



Both x-simple
and y-simple

<u>Example</u> (ii) Find the area bounded by $y = x^2$ and $x + y = 2$.



x-simple so $dA = (x^2 - (2-x))dx$
need limits so find intersection points:

$$\left.\begin{aligned} x^2 &= 2 - x \\ x^2 + x - 2 &= 0 \\ (x+2)(x-1) &= 0 \end{aligned}\right\} \text{ so } x = -2 \text{ or } 1$$

$$A = \int_{-2}^{1} (x^2 + x - 2)\,dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x \Big|_{-2}^{1} = -\frac{9}{2}$$

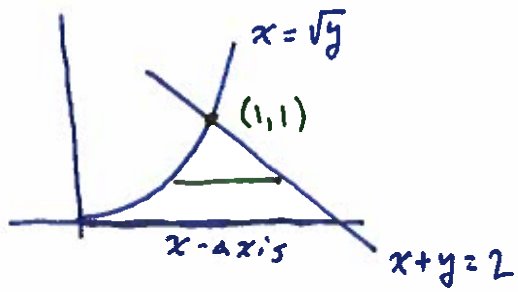
What went wrong?

Answer: the top curve is $y = 2 - x$, the bottom curve is $y = x^2$, so

$$dA = (2 - x - x^2)\,dx \quad \text{and}$$

$$A = \int_{-2}^{1} (2 - x - x^2)\,dx = \frac{9}{2}$$

<u>Example</u> (iii) Find the area bounded by $x = \sqrt{y}$, $x + y = 2$ and the x-axis.



y-simple, not x-simple

so $dA = (2 - y - \sqrt{y})\,dy$

b/c right curve: $x = 2 - y$

left curve: $x = \sqrt{y}$

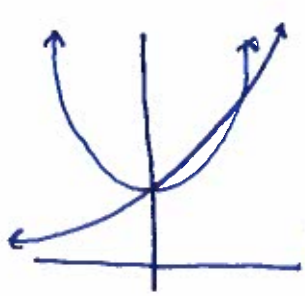
with $0 \le y \le 1$

$$A = \int_0^1 (2 - y - y^{1/2})\,dy$$

$$= 2y - \tfrac{1}{2}y^2 - \tfrac{2}{3}y^{3/2} \Big|_0^1$$

$$= \frac{5}{6}$$

<u>Example</u> (iv) bounded by $y = 2^x$ and $y = \cosh(x)$



Use calculater or Mathematica

$$A = \int_0^{2.22073} (2^x - \cosh(x))\,dx$$

$$= \frac{2^x}{\ln(2)} - \sinh(x) \Big|_0^{2.22073} \approx 0.7294$$