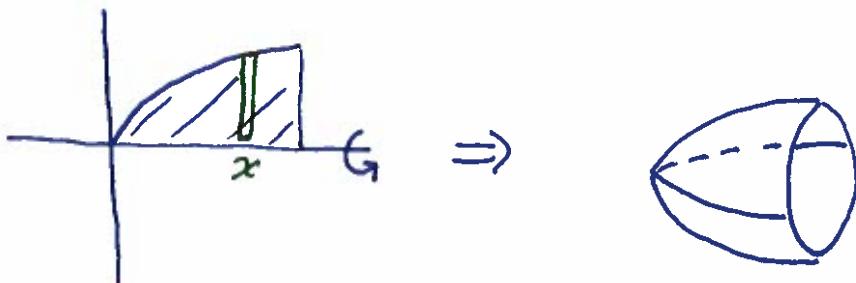


§ 6.2 Volumes (Slicing)

See the book (page 431) for pictures.

We model the volume as the sum of thin slices.

Example (i) Rotate the region bounded by $y = \sqrt{x}$, $x = 2$ and $y = 0$, about the x -axis's



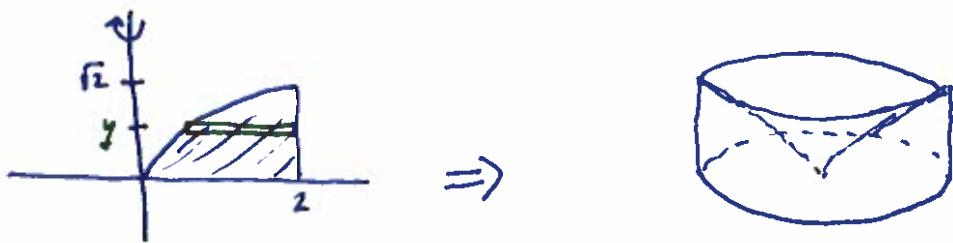
Look at a thin rectangle located at $x \in [0, 2]$ rotated about the x -axis's \Rightarrow disk:

$$\text{disk } \}^R \quad dV = \pi R^2 dx$$

$$\text{where } R = \sqrt{x}$$

so $V = \int_0^2 \pi (\sqrt{x})^2 dx = 2\pi$

Example (ii) Rotate the region bounded by $y = \sqrt{x}$, $x = 2$, $y = 0$, about the y -axis's



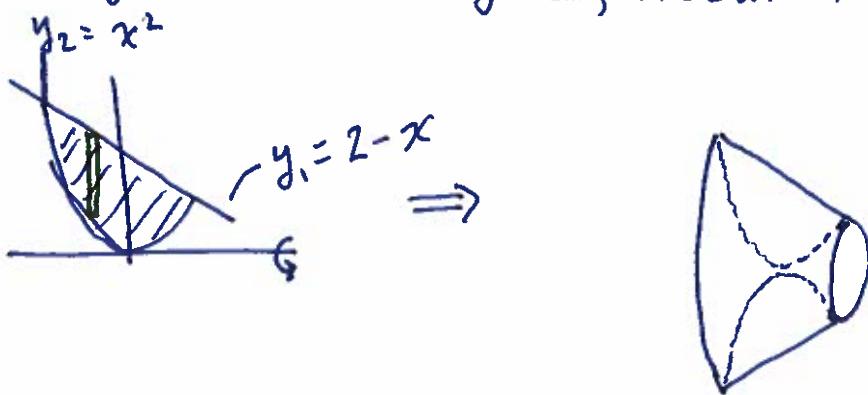
Look at rectangle parallel to x -axis is rotated about the y -axis \Rightarrow washer

$$dV \cancel{\text{is}} = \pi R^2 dy - \pi r^2 dy \\ = \pi (R^2 - r^2) dy$$

$$V = \pi \int_0^{R^2} (2^2 - (y^2)^2) dy \quad \text{where } R = 2, r = y^2 \\ = \frac{16\pi\sqrt{2}}{5} \approx 4.5255$$

Mathematica

Example (iii) Rotate the region bounded by $y = x^2$ and $x + y = 2$, about the x -axis.



Washer:

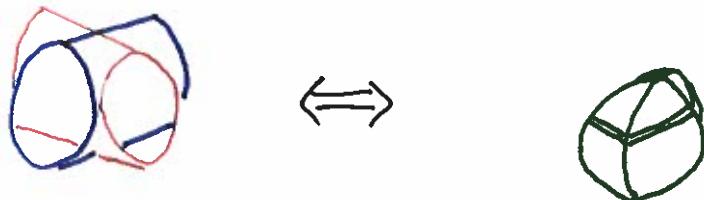
$$\left. \begin{array}{l} R = y_1 = 2 - x \\ r = y_2 = x^2 \end{array} \right\} -2 \leq x \leq 1$$

$$dV = \pi ((2-x)^2 - (x^2)^2) dx$$

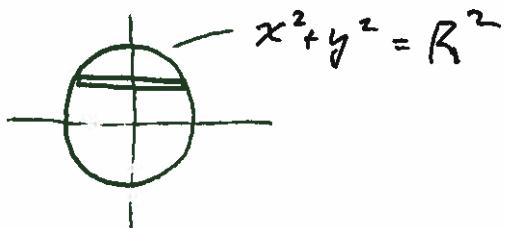
$$V = \pi \int_{-2}^1 (4-4x+x^2-x^4) dx$$

$$= \frac{72\pi}{5} \approx 45.2389$$

Example (iv) Volume generated by the intersection of 2 circular cylinders (of radius R) with axis of symmetry intersecting at right angles.
See problem 64 page 440 of the book.



Side view:



Slice:



$$dV = l^2 dy$$

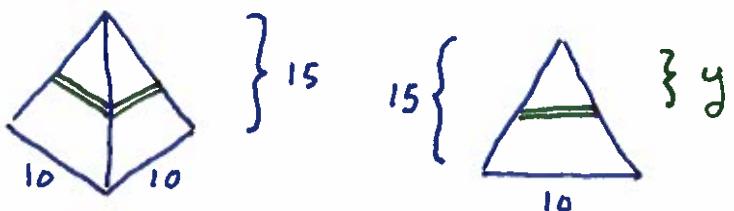
$$\text{where } l = 2\sqrt{R^2 - y^2}$$

$$V = \int_{-R}^R (2\sqrt{R^2 - y^2})^2 dy$$

$$= 8 \int_0^R (R^2 - y^2) dy$$

$$= \frac{16}{3} R^3$$

Example (v) Find the volume of the pyramid with square base if the sides of the base are 10m and the height of the pyramid is 15m.



Slices are square boxes

$$\cancel{dV} = l^2 dy$$



To find l use similar triangles:

$$\frac{l}{10} = \frac{y}{15} \Leftrightarrow l = \frac{2}{3}y$$

So

$$\begin{aligned} V &= \int_0^{15} \left(\frac{2}{3}y\right)^2 dy \\ &= \frac{4}{9} \int_0^{15} y^2 dy \\ &= 500 \end{aligned}$$

(Note: $V = \frac{1}{3}(\text{area of base})(\text{height}) = \frac{1}{3}(100)(15) = 500$)