

## §6.4 Work

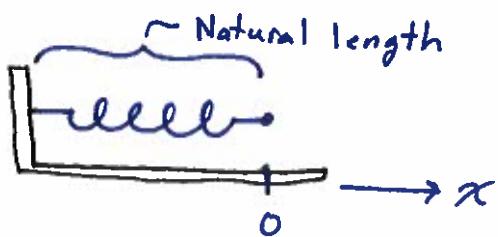
Force = (mass)(acceleration)

$$F = m a = m \frac{d^2 s}{dt^2} \text{ where } s \text{ is position}$$

Work = (Force)(Distance)

$$W = FD$$

Hooke's Law for "perfect" springs



$$\text{Force : } f(x) = kx$$

↑  
spring constant      ↑  
displacement from  
natural length.

Example (i) A spring requires a 10 lb force to displace the spring .25 ft. How much work is required to displace the spring .5 ft from its natural length?

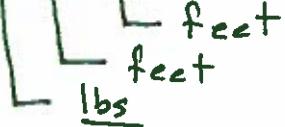
Note: Units of work: foot-pounds or Newton-meters.

The first sentence gives information needed to find the spring constant:

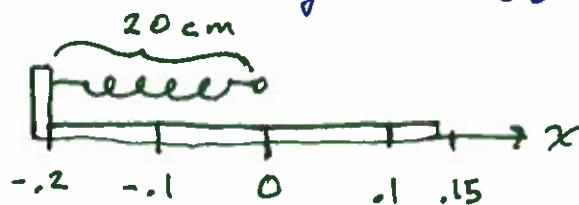
$$f(x) = kx \Leftrightarrow 10 \text{ lb} = (k)(.25 \text{ ft})$$

$$\text{so } k = 40 \frac{\text{lb}}{\text{ft}}$$

$$\text{So, } W = \int_0^5 40x \, dx$$


  
 $= 20x^2 \Big|_0^{1/2}$   
 $= 5 \text{ ft-lbs}$

Example (ii) A 2 Newton force is required to displace a spring from a natural length of 20 cm to a length of 30 cm. How much work is required to stretch the spring from a length of 30 cm to a length of 35 cm?



Natural length of 20 cm = 0 meters on axis

length of 30 cm = .1 meters displacement

length of 35 cm = .15 meter displacement

Find spring constant:

$$2 = (k)(.1) \Rightarrow k = 20 \frac{\text{N}}{\text{m}}$$

then

$$W = \int_{.1}^{.15} 20x \, dx = \frac{1}{8} \text{ N-m}$$

Any time a force is applied to a moving object, work is done.

For the spring, as the displacement changes, the force changes. Thus

$$dW = f(x) dx$$

↑ small displacement

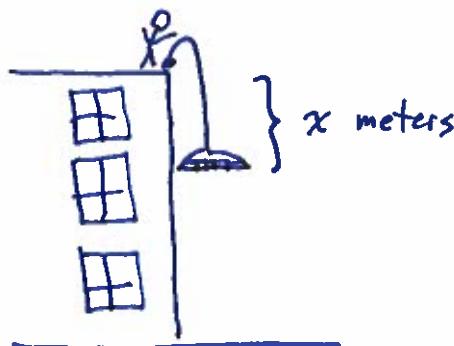
We may also have situations where

$$dW = D \cdot dF$$

↑ small "slice" of force  
displacement

Example (iii) is another case where  $dW = f(x) dx$ .

Example (iii) A flat of bricks has a mass of 50 kg and is attached to a chain with a density of  $\frac{1}{2}$  kg/meter. How much work is required to pull the bricks (and chain) from ground level to the top of a building that is 30 meters tall?



$$F = -9.8 \underbrace{\left(50 + \frac{1}{2}x\right)}_{\text{mass}}$$

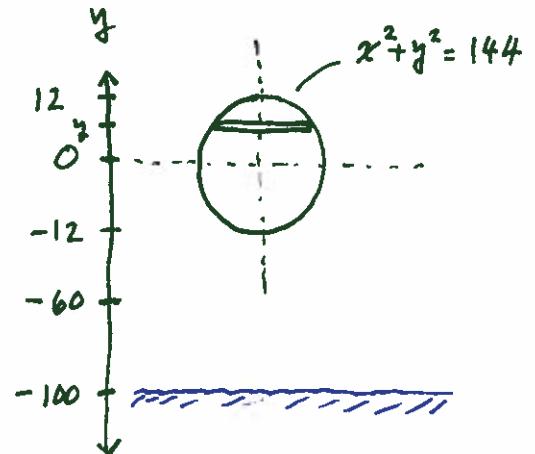
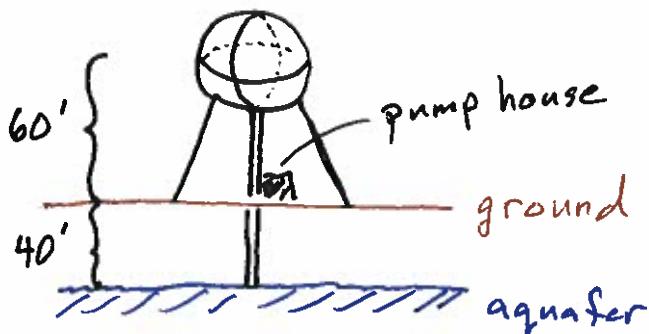
↑ gravity

$D = dx$  b/c  $F$  changes based on length of chain

$$W = F \cdot D = -9.8 \int_{30}^0 \left(50 + \frac{1}{2}x\right) dx$$
$$= 16905 \text{ N}\cdot\text{m}$$

Now an example where  $dW = D \cdot dF$

Example (iv) A spherical tank of radius 12 ft is on a tower 60 feet (from the center of the tank) above the ground. How much work is required to fill the tank (from empty) by moving water in an aquifer 40 feet below ground to the tank?



We fill the tank by moving one "slice" (Disk) at a time. The volume of the slice is

$$dV = \pi (\sqrt{144 - y^2})^2 dy \quad \text{disk method sec. 6.2}$$

The density of water is 62.5 lb/ft<sup>3</sup> or 9800 N/m<sup>3</sup>,

$$dF = 62.5 dV$$

$$D = y + 100$$

so  $W = 62.5 \pi \int_{-12}^{12} (y+100)(144-y^2) dy$

$$= 4.524 \times 10^7 \text{ ft-lbs}$$