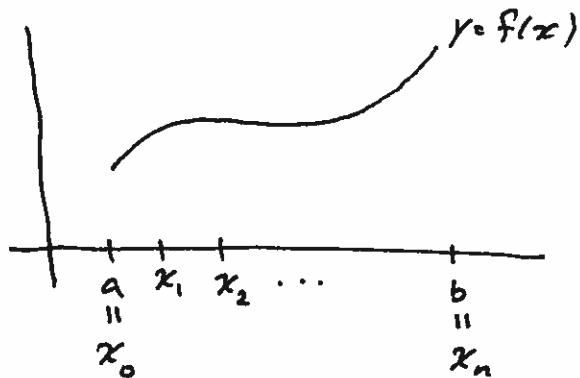


§6.5 Average Value of a Function

$$\text{Average} = \frac{y_1 + y_2 + \dots + y_n}{n}$$



$$\begin{aligned}\text{avg } (f) &= \frac{f(x_1) + f(x_2) + \dots + f(x_n)}{n} \\ &= \frac{f(x_1) + f(x_2) + \dots + f(x_n)}{b-a} \cdot \frac{b-a}{n} \\ &= \frac{1}{b-a} \sum_{k=1}^n f(x_k) \Delta x\end{aligned}$$

Now let $n \rightarrow \infty$ to get

$$\text{Avg}_{[a,b]}(f) = \frac{1}{b-a} \int_a^b f(x) dx$$

provided the integral exists.

The Mean Value Theorem

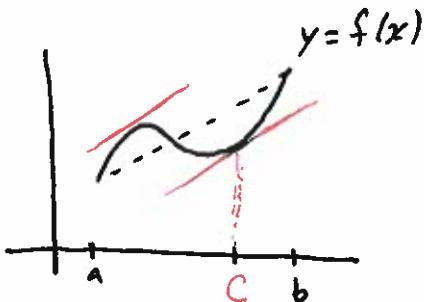
For derivatives: If f is continuous on $[a,b]$ and differentiable on (a,b) , then there is a number $c \in (a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

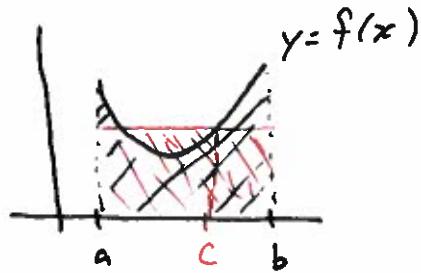
For integrals: If f is continuous on $[a,b]$, then there is a number $c \in [a,b]$ such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx = \text{Avg}_{[a,b]}(f).$$

The Mean Value Theorem for derivatives and for integrals are really the same.



For derivatives:
exists tangent parallel
to secant through
the endpoints



For integrals:
area of rectangle
is equal to area
under the curve

Example Find the average value of $f(x) = e^{2x}$ on $[-1, 1]$, then find a number $c \in [-1, 1]$ satisfying the MVT for integrals.

$$\begin{aligned}\text{Avg}_{[-1,1]}(f) &= \frac{1}{1 - (-1)} \int_{-1}^1 e^{2x} dx \\ &= \frac{1}{2} \cdot \frac{1}{2} e^{2x} \Big|_{-1}^1 \\ &= \frac{1}{4} (e^2 - e^{-2}) \approx 1.8134\end{aligned}$$

then

$$\begin{aligned}f(c) &= \frac{1}{4} (e^2 - e^{-2}) \\ \Leftrightarrow e^{2c} &= \frac{1}{4} (e^2 - e^{-2}) \\ 2c &= \ln\left(\frac{e^2 - e^{-2}}{4}\right) \\ c &= \frac{1}{2} \ln\left(\frac{e^2 - e^{-2}}{4}\right) \approx .2976\end{aligned}$$

