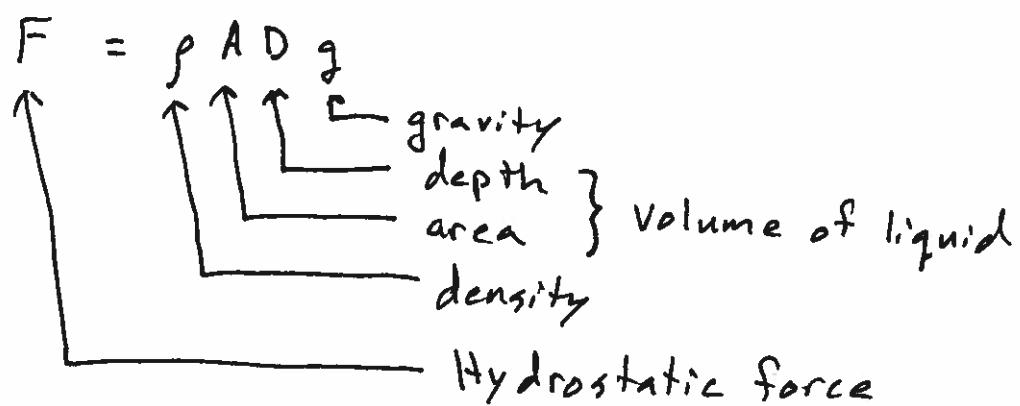
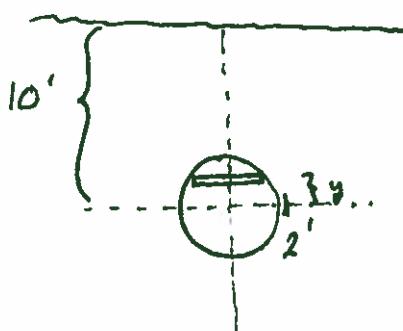


§8.3 Applications to Physics and Engineering

$$\text{Force} = (\text{mass})(\text{acceleration})$$



Example (i) Find the hydrostatic force on one end of a cylindrical drum with radius 2 ft if it is submerged in water 10 feet deep (at its center).



$$\text{For water } \rho g = 62.5 \text{ lbs/ft}^3$$

$$\text{Equation of circle: } x^2 + y^2 = 4$$

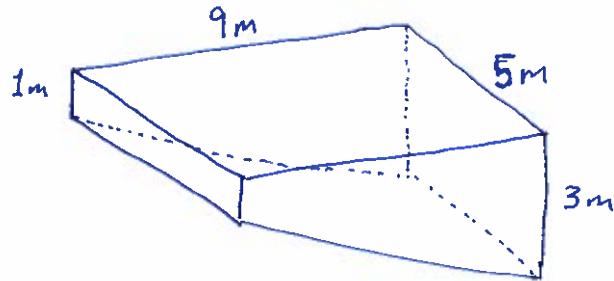
$$dF = \rho g (\text{depth}) dA$$

$$= 62.5 (10-y) (2\sqrt{4-y^2}) dy$$

$$F = \int_{-2}^2 125 (10-y) \sqrt{4-y^2} dy$$

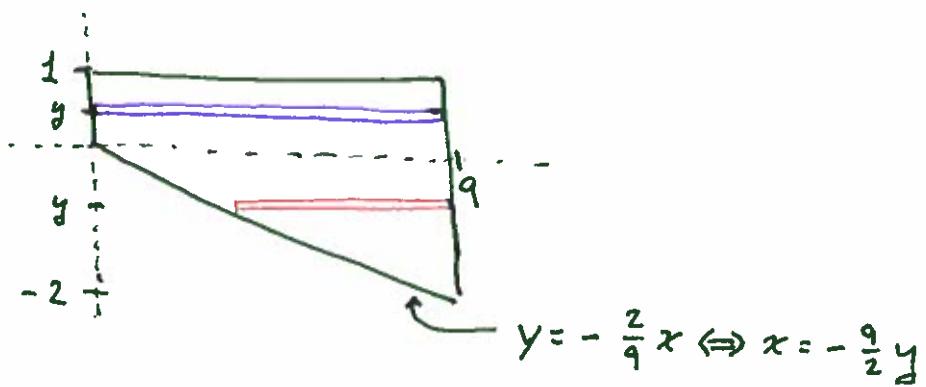
$$= 2500 \pi \approx 7853.98 \text{ lbs}$$

Example (ii) Swimming pool: Full of water.



For water $\rho g = 9800 \text{ N/m}^3$. Hydrostatic force for left and right side are easy.

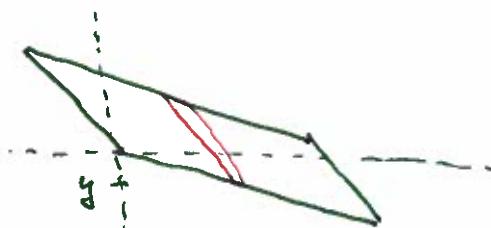
Front/Back sides:



depth = $1 - y$ for both regions

$$\begin{aligned} F &= 9800 \left(\int_0^1 (1-y)(9)dy + \int_{-2}^0 (1-y)\left(9 + \frac{9}{2}y\right)dy \right) \\ &\equiv 191,100 \text{ N} \end{aligned}$$

Bottom:

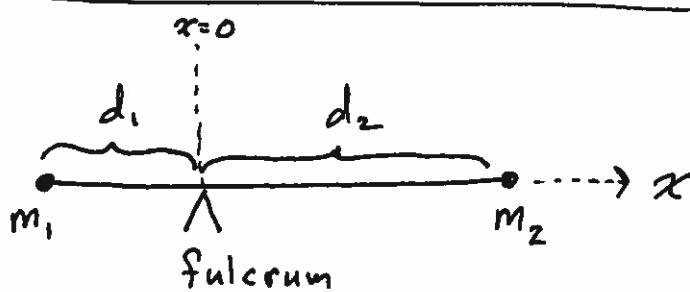


depth = $1 - y$ as above

$$dA = 5 ds = 5\sqrt{1 + \left(-\frac{9}{2}\right)^2} dy$$

$$F = 9800 \int_{-2}^2 (1-y)\left(5\sqrt{1 + \frac{81}{4}}\right)dy \equiv 9800\sqrt{85}$$

Mass, Moments & Center of mass



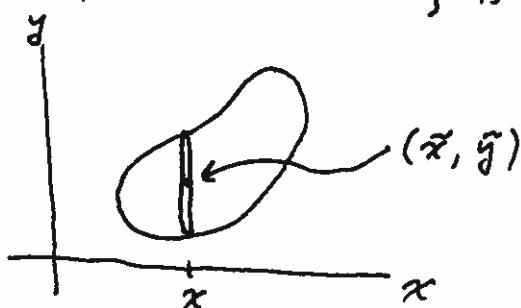
$$\text{Mass} = m = \sum m_k = m_1 + m_2$$

$$\text{Moment} = M = \sum d_k m_k = d_2 m_2 - d_1 m_1$$

$$\text{Center of mass} = \bar{x} = \frac{M}{m}$$

For the system shown, if $d_1 m_1 = d_2 m_2$, then the center of mass is $\bar{x} = 0$. The system is balanced at the fulcrum.

2D: Thin plates where g is a function of x .



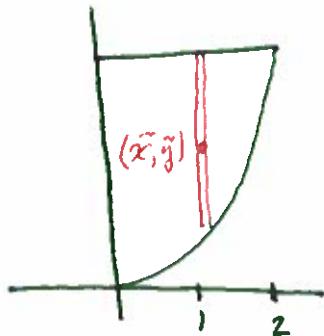
We consider the entire mass of the strip to be located at the centroid of the strip, labeled (\bar{x}, \bar{y}) . Then

$$dm = g(x)dx, \quad dM_y = \bar{x} dm, \quad dM_x = \bar{y} dm$$

We integrate to find m , M_y and M_x , then

$$\bar{x} = \frac{M_y}{m} \quad \text{and} \quad \bar{y} = \frac{M_x}{m}.$$

Example (iii) Find the center of mass of the thin plate bounded by the graphs of $y = x^2$, $y = 4$, $x \geq 0$, with density given by $\rho(x) = x + 1$.



$$\tilde{x} = x, \quad \tilde{y} = \frac{4+x^2}{2}$$

$$dm = (x+1)(4-x^2)dx$$

$$dM_y = \tilde{x} dm$$

$$dM_x = \tilde{y} dm$$

$$m = \int_0^2 (x+1)(4-x^2)dx = \frac{28}{3}$$

$$M_y = \int_0^2 x(x+1)(4-x^2)dx = \frac{124}{15}$$

$$M_x = \int_0^2 \left(\frac{4+x^2}{2}\right)(x+1)(4-x^2)dx = \frac{352}{15}$$

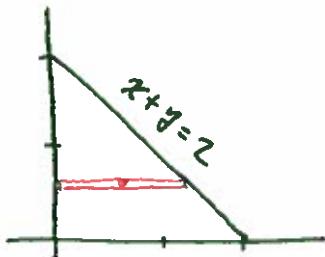
$$\bar{x} = \frac{M_y}{m} = \frac{31}{35} \approx 0.8857$$

$$\bar{y} = \frac{M_x}{m} = 2.5143$$

This point seems reasonable.

A centroid is the geometric center ($\rho(x)=1$).

Example Find the centroid and center of mass of the triangle given by $x \geq 0$, $y \geq 0$ and $y \leq -x+2$, if the density is given by $\rho(y) = e^y$.



$$\bar{x} = \cancel{0}, \bar{y} = y$$

$$\bar{x} = \frac{2-y}{2}$$

Centroid: $m = \int_0^2 (2-y) dy = 2$

$$M_y = \int_0^2 \left(\frac{2-y}{2}\right)(2-y) dy = \frac{4}{3}$$

$$M_x = \int_0^2 y(2-y) dy = \frac{4}{3}$$

Note: we could see $M_x = M_y$ by symmetry

centroid: $(\frac{2}{3}, \frac{2}{3})$

C.O.M.

$$m = \int_0^2 e^y (2-y) dy = e^2 - 3$$

$$M_y = \int_0^2 e^y \left(\frac{2-y}{2}\right)(2-y) dy = e^2 - 5$$

$$M_x = \int_0^2 e^y \cdot y(2-y) dy = 4$$

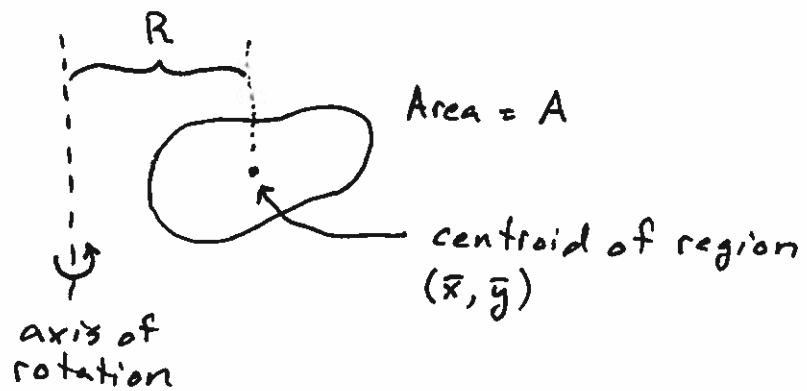
$$(\bar{x}, \bar{y}) = (.5443, .9114)$$

Note: centroid of any triangle



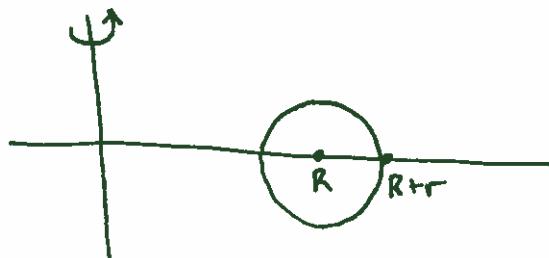
Intersection of medians ($\frac{1}{3}$ of the way from the midpoint to opposite vertex).

Theorem of Pappus



$$V = 2\pi R A$$

Example (iv) Find the volume of a torus of little radius r and big radius R .



$$A = \pi r^2$$

$$\begin{aligned} V &= (2\pi R)(\pi r^2) \\ &= 2\pi^2 r^2 R \end{aligned}$$