# MATH 333
## Lab04ConstrainedOp

*Objective*
The objective of this project is to use *Mathematica* to explore some of the concepts behind constrained function optimization using Lagrange Multipliers.

*Narrative*
If you have not already done so, read Section 15.8 in the text. In this section we learn that finding the extrema of the function $z = f(x, y)$ subject to the constraint $g(x, y) = 0$, can be reduced to solving the system of 3 equations

$$\nabla f(x, y) = \lambda \nabla g(x, y), \; g(x, y) = 0$$

in the 3 unknowns $x$, $y$, and $\lambda$. In this project we consider the specific problem of finding the extrema of

$$f(x, y) = 3x^2 - xy + y^2 - 3x - 5y + 9$$

subject to the sonstraint (along the curve $C$ whose equation is)

$$x^2 + y^2 = 1$$

*Task*

(1) Type the following lines into *Mathematica*.

```
(* Project 04: Constrained Optimization*)
f[x_, y_] = 3*x^2 - x*y + y^2 - 3*x - 5*y + 9
plot0 = ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2},
  Contours -> {0, 3, 6, 9, 12, 15, 18, 21, 24},
  ContourShading -> None, ContourStyle -> Directive[Red, Thick]];
plot1 = ContourPlot[x^2 + y^2 == 1, {x, -2, 2}, {y, -2, 2},
  ContourStyle -> Directive[Blue, Thick]];
Show[plot0, plot1]
```

At this time, make a hard-copy of your typed input and *Mathematica's* responses. Then, ...

(2) By hand, carefully draw the level curves of $f$ that correspond to the extreme values of $f$ along $C$. Label each of the curves you have drawn by hand with the value of $f$ along it. (To find the value of $f$ along each curve, first *estimate* the coordinates of the point of intersection of each level curve with $C$, and $f$; all you need is an estimate: you do *not* have to solve the Lagrange multiplier system of equations! Then use *Mathematica* to substitute the coordinates of each point of intersection into $f$.)

(3) Label the level curves on each side of each of the level curves you have drawn by hand with the value of $f$ along it. (Use the same technique you used in part (2) to obtain these values.)

(4) At each of the points where $C$ meets these two level curves, draw and label the tangent line to $C$ and a vector in the direction of the gradient $\nabla f$ of $f$. (Remember: The direction of the gradient is the direction of *steepest ascent*.)

*Comments*

There are often ways to approach constrained optimization problems other than Lagrange multipliers. For example, the problem addressed above can be reduced to a problem in the optimization of a single function of a single variable by observing that if we parametrize the curve $x^2 + y^2 = 1$ by $x(t) = \cos(t)$, $y(t) = \sin(t)$, $t \in [0, 2\pi]$ then, along the graph of $g(x, y) = 0$,

$$f = f(t) = \cos^2(t) - \cos(t)\sin(t) + 3\sin^2(t) - 5\cos(t) - 3\sin(t) + 9.$$