MATH 333

Lab05DoubleIntegrals

Objective The objective of this project is to illustrate how *Mathematica* can be used to evaluate double integrals.

Narrative If you have not already done so, read Section 15.1-15.3 of the text. In this project we illustrate how *Mathematica* can be used to evaluate double integrals.

Task

(1) Integrals over rectangular regions.

(a) Type the command lines below into *Mathematica* in the order in which they are listed; they draw the rectangle R bounded by the lines $x = 1$, $x = 3$, $y = 2$, $y = 4$, and evaluates the integral $\int_2^4 \int_1^3 x^4 y^3 e^{-xy}\,dx\,dy$.

```
(* Parts 1a and 1b *)
ContourPlot[{x == 1, x == 3, y == 2, y == 4}, {x, 0, 4},
   {y, 0, 5}, ContourStyle -> Directive[Blue, Thick]]
f[x_, y_] = x^4*y^3*Exp[-x*y];
int1a = Integrate[Integrate[f[x, y], {x, 1, 3}], {y, 2, 4}]
```

(Would you want to compute the integral by hand? Probably not. This is one of the things that makes *Mathematica* quite usefull.)

(b) Type the command line necessary to compute $\int_1^3 \int_2^4 x^4 y^3 e^{-xy}\,dy\,dx$. Observe that we get the same value for the integral irrespective of the order of the limits. While this *always* happens when R is a rectangle it does *not* always happen when R is not a rectangle (as we will see below).

(2) Integrals over non-rectangular regions.

(a) Type the command lines below into *Mathematica*; they find the points of intersection of $y = 2x^2$ and $y = 1 + x^2$, draw the region R bounded by these two curves, and evaluates $\int\int_R (x + 2y)\,dy\,dx$.

```
(* Part 2a and 2b *)
intersectionpts = Solve[{y == 2*x^2, y == 1 + x^2}, {x, y}]
ContourPlot[{y == 2*x^2, y == 1 + x^2}, {x, -1, 1},
   {y, 0, 3}, ContourStyle -> Directive[Blue, Thick]]
f[x_, y_] = x + 2*y;
int2a = Integrate[Integrate[f[x, y], {y, 2*x^2, 1 + x^2}],
   {x, -1, 1}]
```

(b) Continue by typing the command line below into *Mathematica*. It evaluates the integral where R is the region of part (c), *intentionally using the wrong order of integration*. Note that not only does our answer depend on the order of integration, but when we integrate in the wrong order our “answer” isn’t even a real number!

```
int2b = Integrate[Integrate[f[x, y], {x, -1, 1}],
   {y, 2*x^2, 1 + x^2}]
```

(c) Type the command lines necessary to find the points of intersection of the region R bounded by $x = 1 - y^2$ and $x = y - y^3$, draw the region bounded by these two curves and evaluate the integral $\int\int_R (x^2 - 4y)\, dA$. (*Hint*: Look at parts (a) and (b) of Task 2.)

At this point, make a hard-copy of your typed input and *Mathematica's* responses. Then, by hand, label the curves bounding the region with their equations, and shade in the region.