MATH 333
Lab06RectCylSphCoord

Objective
The objective of this project is to better acquaint you with cylindrical and spherical coordinates.

Narrative
In this project we use the **ParametricPlot3D** command in *Mathematica* along with the change of coordinate formulas to graph basic surfaces in rectangular, cylindrical, and spherical coordinates. *Mathematica* has a built in command to graph surfaves in spherical coordinates, but not in cylidrical, so we will stay with the more basic **ParametricPlot3D** command.

Task

(1) We begin by creating some graphics. (Note: Although a default set of display options is specified, you may, depending on the hardware youre using, need to modify them by manually rotating the graphics to get an acceptable hard-copy.)

(a) First we consider rectangular coordinates. Type the command lines below into *Mathematica* in the order in which they are listed.

```
(* Part a: Rectangular Coordinates *)
ParametricPlot3D[{1, y, z}, {y, 0, 1}, {z, 0, 1},
   PlotRange -> {{-.5, 1.5}, {-.5, 1.5}, {-.5, 1.5}}]
ParametricPlot3D[{x, 1, z}, {x, 0, 1}, {z, 0, 1},
   PlotRange -> {{-.5, 1.5}, {-.5, 1.5}, {-.5, 1.5}}]
ParametricPlot3D[{x, y, 1}, {x, 0, 1}, {y, 0, 1},
   PlotRange -> {{-.5, 1.5}, {-.5, 1.5}, {-.5, 1.5}}]
```

Note that these graphs represent part of the planes $x = 1$, $y = 1$, and $z = 1$.

(b) Next we consider cylindrical coordinates. Type the command lines below into *Mathematica* in the order in which they are listed. Note that the θ may be found on the Math Assist palette.

```
Clear[x, y]
x[r_, \[Theta]_] = r*Cos[\[Theta]]
y[r_, \[Theta]_] = r*Sin[\[Theta]]
ParametricPlot3D[{x[1, \[Theta]], y[1, \[Theta]], z},
   {\[Theta], 0, 2*Pi}, {z, 0, 2},
   PlotRange -> {{-1.5, 1.5}, {-1.5, 1.5}, {-.5, 2.5}}]
ParametricPlot3D[{x[r, 3*Pi/4], y[r, 3*Pi/4], z}, {r, -1, 1},
   {z, 0, 2}, PlotRange -> {{-1.5, 1.5}, {-1.5, 1.5},
   {-.5, 2.5}}]
ParametricPlot3D[{x[r, \[Theta]], y[r, \[Theta]], 1},
   {\[Theta], 0, 2*Pi}, {r, 0, 1},
 PlotRange -> {{-1.5, 1.5}, {-1.5, 1.5}, {-.5, 2.5}}]
```

(c) Next we consider spherical coordinates. Type the command lines below into *Mathematica* in the order in which they are listed. Note that the ρ, θ, and ϕ may be found on the Math Assist palette.

```
Clear[x, y, z]
x[\[Rho]_, \[Theta]_, \[Phi]_] = \[Rho]*Sin[\[Phi]]*Cos[\[Theta]]
y[\[Rho]_, \[Theta]_, \[Phi]_] = \[Rho]*Sin[\[Phi]]*Sin[\[Theta]]
z[\[Rho]_, \[Theta]_, \[Phi]_] = \[Rho]*Cos[\[Phi]]
ParametricPlot3D[{x[1, \[Theta], \[Phi]], y[1, \[Theta], \[Phi]],
   z[1, \[Theta], \[Phi]]}, {\[Theta], 0, 2*Pi},
   {\[Phi], Pi/3, Pi}, PlotRange -> {{-1.5, 1.5}, {-1.5, 1.5},
   {-1.5, 1.5}}]
ParametricPlot3D[{x[\[Rho], 3*Pi/4, \[Phi]],
   y[\[Rho], 3*Pi/4, \[Phi]], z[\[Rho], 3*Pi/4, \[Phi]]},
   {\[Rho], 0, 1}, {\[Phi], 0, Pi/2},
   PlotRange -> {{-1.5, 1.5}, {-1.5, 1.5}, {-1.5, 1.5}}]
ParametricPlot3D[{x[\[Rho], \[Theta], Pi/4],
   y[\[Rho], \[Theta], Pi/4], z[\[Rho], \[Theta], Pi/4]},
   {\[Rho], 0, 1}, {\[Theta], 0, 2*Pi},
   PlotRange -> {{-1.5, 1.5}, {-1.5, 1.5}, {-1.5, 1.5}}]
```

At this time make a hard-copy of your typed input and *Mathematicas* responses (both text and graphics). Then, ...

(2) Label the graphics you produced as follows:
   (a) On each graphic you produced in part (a) of Task 1, label the positive $x-$, $y-$, and $z-$ coordinate directions.
   (b) On each graphic you produced in part (b) of Task 1, label the positive $x-$, $y-$, and $z-$ coordinate directions. On the first graphic, highlight by hand that part of the surface over which θ is between 0 and $\pi/2$, on the second highlight that part over which r is between 0 and 1, and on the third that part over which r is between 0.5 and 1 and θ is between 0 and $\pi/2$.
   (c) On each graphic you produced in part (c) of Task 1, label the positive $x-$, $y-$, and $z-$ coordinate directions. On the first graphic, highlight by hand that part of the surface over which θ is between 0 and $\pi/2$ and ϕ is between $\pi/4$ and $\pi/2$, on the second highlight that part over which ϕ is between $\pi/4$ and $\pi/2$, and on the third that part over which θ is between 0 and $\pi/2$.

Comments

(1) The parameters θ and ϕ you were adjusting in Project 13.5b were the θ and ϕ spherical coordinates of the viewer!
(2) The effect of replacing the 1 in the [x,y,1] command line of Task 1(a) by a function $z = z(x, y)$ of x and y is to graph $z = z(x, y)$ in rectangular coordinates over the square $x = 0..1$, $y = 0..1$. (As it stands, $z = z(x, y) = 1$.) You can obtain some interesting surfaces if, in the same vein, you replace:
   (a) the 1 in the [1,theta,z] in Task 1(b) with a function $r = r(\theta, z)$,
   (b) the 3*Pi/4 in the [r,3*Pi/4,z] in Task 1(b) with a function $\theta = \theta(r, z)$,
   (c) the 1 in the [r,theta,1] in Task 1(b) with a function $z = z(r, \theta)$,
   (d) the 1 in the [1,theta,phi] in Task 1(c) with a function $\rho = \rho(\theta, \phi)$,
   (e) the 3*Pi/4 in the [rho,3*Pi/4,z] in Task 1(c) with a function $\theta = \theta(\rho, \phi)$,
   (f) the 1 in the [r,theta,1] in Task 1(c) with a function $\phi = \phi(\rho, \theta)$.

Try experimenting, and see what you get!