Name:
PIN:

Applied Matrix Algebra
Eigenvalues and Eigenvectors

Objective The objective of this project is to illustrate how *Mathematica* can be used to calculate eigenvalues and eigenvectors.

Narrative Recall that λ is defined to be an eigenvalue of a square matrix A if there is a vector $\vec{v}$ such that

$$A\vec{v} = \lambda\vec{v}$$

and in this case $\vec{v}$ is called a corresponding eigenvector of A.

Now let's see how *Mathematica* can be used to find these eigenvalues and eigenvectors. Throughout this lab, remember that you can use the symbol λ by accessing the sybol assist tool.

Recall in class that we talked about the characteristic polynomial of a matrix,

$$P(\lambda) = \det(A - \lambda I),$$

and how the eigenvalues are the zeros or roots of this polynomial. Type the commands below into *Mathematica* to define a matrix and find it's characteristic polynomial.

```
A = {{7, 0, 3}, {2, 7, 1}, {0, 0, -10}};
MatrixForm[A]
P = CharacteristicPolynomial[A, \[Lambda]]
```

If we use the solve command, we can find the roots of this polynomial.

```
Solve[P == 0, \[Lambda]]
```

which should return the values: $-10, 7, 7$.

Now let's use some of the other commands in *Mathematica* and see what happens.

```
evals = Eigenvalues[A]
```

Notice that this returns a list whose components are the eigenvalues of A. To get at the individual values we could do the following:

```
evals[[1]]
evals[[2]]
evals[[3]]
```

Notice in the *Mathematica* help the command - **Eigensystem**. Lets see what this does for us.

```
estuff = Eigensystem[A]
```

Notice that this command gives the eigenvalue and a set of vectors corresponding to those eigenvalues. The multiplicity of an eigenvalue is defined as the multiplicity of the corresponding factor in the characteristic polynomial. In this case the eigenvalue 7 has multiplicity 2. In the case of the eigenvalue 7, we have a degenerate eigenvector. In order to get the "actual" set of eigenvectors we need to get the generalized eigenvectors as follows

```
generalestuff = Eigensystem[{N[A], IdentityMatrix[3]}]
```

From this we see that the eigenvectors corresponding to the eigenvalue 7 are {{ 0, 1, 0},{ 0, -1, 0}}. Notice also that the eigenvectors in this case have been normalized (magnitude one).

Task Use the above commands to (i) generate the characteristic polynomial and find its zeros, and (ii) find the eigenvalues, their multiplicity, and the corresponding eigenvector(s) of the given matrices.

(1)

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

(2)

$$M = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

(3)

$$M = \begin{bmatrix} 7 & -9 & -15 \\ 0 & 4 & 0 \\ 3 & -9 & -11 \end{bmatrix}$$

(4)

$$M = \begin{bmatrix} 3 & 2 & 3 & -1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$