§2.6 Numerical Measure of Relative Standing

Here we look at percentile ranking or percentile score

Definition For any set of n measurements (arranged in ascending or descending order), the p^{Th}-percentile is a number such that $p\%$ of the measurements fall below that number and $(100-p)\%$ fall above it.

Note: sometimes high measurements are "good". In this case we arrange the values in ascending order. Sometimes low measurements are "good", the we use descending order.

Example We buy ten apples at a store and measure the weights of each apple. Find the percentile rankings for each

Weights of apples in oz

4.8, 4.9, 5.2, 5.2, 5.3, 5.7, 6.0, 6.1, 6.1, 6.3

The "smallest" apple weighs 4.8 oz. Since there are only 10 apples, we can say that a weight of 4.8 is at the 10^{Th}-percentile rank or the 0^{Th}-percentile rank. The definition is a little ambiguous. We will "put the equal sign" on the "fall above" part of the definition. Thus,

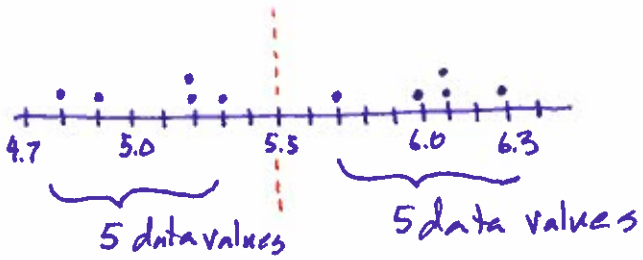
weight	4.8	4.9	5.2	5.3	5.7	6.0	6.1	6.3
%-rank	0	10	20	40	50	60	70	90

Often we want to know the middle score and the middle of the top and bottom half of the data. These numbers are called the quartiles.

Definition the lower quartile (Q_L or Q_1) is the 25^{th}-percentile of the data set. The middle quartile (M or Q_2) is the 50^{th}-percentile. The upper quartile (Q_U or Q_3) is the 75^{th}-percentile.

Example For the apple data, M = 5.7 and, by definition, we don't have values for Q_L and Q_U.

Here is another way of thinking about quartiles: look at a dot Graph for the above data,



Use the median as the middle quartile, $M = \frac{5.3+5.7}{2}$ = 5.5 (the red line value)

For Q_L, $\frac{n}{4} = \frac{10}{4} = 2.5$, so we look at the 3rd data point (or the median of the lower half of the data)

Thus $Q_L = 5.3$. Similarly, $Q_U = 6.1$.

Example Suppose that 41 people in a classroom participate in a quiz. The possible scores are $0, 1, 2, \ldots, 10$. If the frequency chart for the scores the 41 people earned is

score	0	1	2	3	4	5	6	7	8	9	10
freq.	0	2	0	1	2	4	8	15	4	3	2

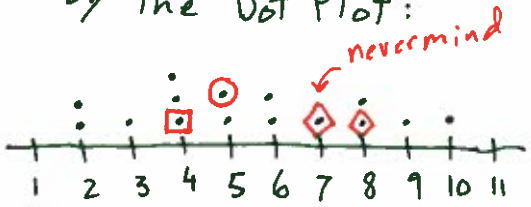
Find Q_L, M and Q_U.

First: with 41 data points, the median is the 21^{ST} score. So $M = 7$.

Next: there are 20 scores "to the left" of M. Thus the median of these 20 scores is the average of the 10^{Th} and 11^{Th} data points. So $Q_L = \frac{6+6}{2} = 6$.

Finally: there are 20 scores "to the right" of M. Thus, the median of these 20 scores is the average of the 10^{Th} and 11^{Th} of these scores, or the 31^{ST} and 32^{nd} overall scores. So $Q_U = \frac{7+7}{2} = 7$.

Example Find Q_L, M and Q_U for the following data given by the Dot Plot:



$n = 15$

The median score is the 8^{Th} one indicated by the ○. So $M = 5$. There are seven data points "above" and "below", so the 4^{Th} data point is $Q_L = 4$ (□). The 12^{Th} data point is $Q_U = 8$ (◇)