# §2.6 (Part 2) z-scores

Sample z-score for a measurement $x$:

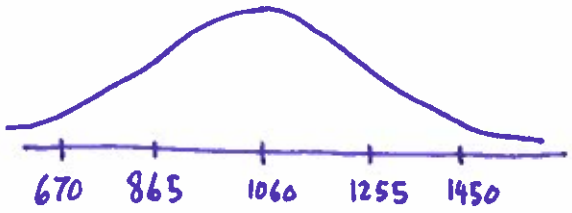
$$z = \frac{x - \bar{x}}{s}$$

Population z-score for a measurement $x$:

$$z = \frac{x - \mu}{\sigma}$$

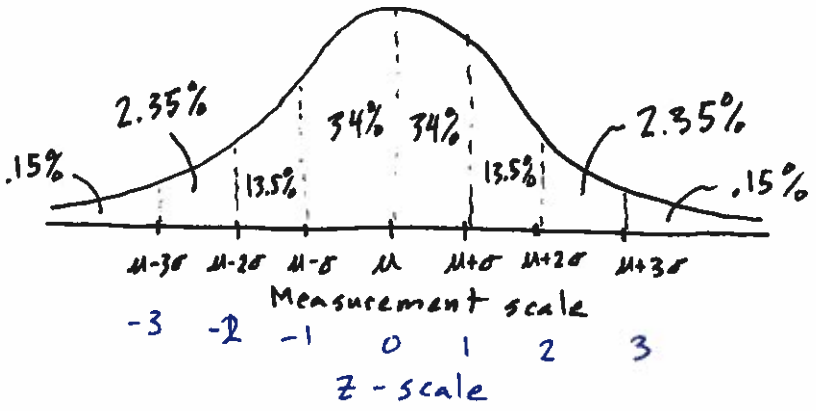
Example In 2017, for the United States, the SAT had a mean of 1060 and a standard deviation of 195. Calculate the z-score for $x = 1400$. Interpret this result in terms of the Empirical Rule.

$$z = \frac{x - \mu}{\sigma} = \frac{1400 - 1060}{195} = 1.74$$




This z-score means a 1400 is between one and two standard deviations above the mean.

z-scores are a way of "standardizing" data that has a mound shaped, symmetric PDF.




Example A statistics exam has a mean of 75 and a standard deviation of 8. If a studend had a z-score of 1.625, then what was their actual score on the exam?

$$z = \frac{x - \bar{x}}{s} \iff x = z \cdot s + \bar{x}$$

So $x = (1.625)(8) + 75 = 88$

the student scored an 88 on the exam.

Approximately what percentage of the students scored bellow 67 on the exam and what z-score correspond to a 67 on this exam?

$$z = \frac{67 - 75}{8} = -1$$

In the Empirical Rule, the area under the curve for $z < -1$ is $13.5\% + \underbrace{2.35\% + .15\%}_{2.5\%} = 16\%$

So approximately 16% of the students scored below a 67 on the exam, based on the Empirical Rule.

Example A student at a large university is enrolled in both a calculus III course and a differential equations course. Both classes have a large number of students and the instructors for both classes have stated that the classes will be graded on a curve. On the first exam in calc. III, the student scored an 83 where the class average was 67.8 and standard deviation was 7.2. In the D.E. class, it's first exam had a mean of 70.6, standard deviation of 9.4 and that same student scored an 88. In which class did the student get a higher grade relative to the other students in the courses?

For calc III: $\bar{x} = 67.8$, $s = 7.2$, $x = 83$. So

$$z = \frac{83 - 67.8}{7.2} = 2.111$$

For D.E.: $\bar{x} = 70.6$, $s = 9.4$, $x = 88$. So

$$z = \frac{88 - 70.6}{9.4} = 1.851$$

Thus the student actually did better in calc III.