

## §4.2 Probability Distributions for Discrete Random Variables

Example Rolling two, 6-sided dice and adding the dots.

Assuming the dice are "fair", the probability of any one of the sided being face up is the same. There are 36 possible outcomes:  $\square \square, \square \square, \dots, \square \square, \square \square, \square \square, \dots, \square \square, \dots, \square \square$ .

Sum	ways	Probability
2	1+1	$\frac{1}{36}$
3	1+2, 2+1	$\frac{2}{36} = \frac{1}{18}$
4	1+3, 2+2, 3+1	$\frac{3}{36} = \frac{1}{12}$
⋮		
7	1+6, 2+5, 3+4, 4+3, 5+2, 1+6	$\frac{6}{36} = \frac{1}{6}$
8	2+6, 3+5, 4+4, 5+3, 6+2	$\frac{5}{36}$
⋮		
12	6+6	$\frac{1}{36}$

The missing rows are left for the student to figure out.

Definition The probability distribution of a random variable is a graph, table, chart, or formula that specifies the probability associated with each possible value of that random variable.

Requirements for the probability distribution of a discrete random variable.

(1)  $P(x) \geq 0$  for all  $x$ .

(2)  $\sum P(x) = 1$

where the sum is over all possible values of  $x$ .

Example A frequency diagram for student quiz scores is given. Generate the PDF for this data and find the probability of selecting a student at random ~~and~~ who scored an 8 out of 10 on the quiz? What is the probability that a randomly selected student scored a 9 or 10 on the quiz?

score	0	1	2	3	4	5	6	7	8	9	10
freq.	0	0	1	0	3	6	8	10	7	5	2

$N = 42$ . The relative frequency is the PDF:

score	0	1	2	3	4	5	6	7	8	9	10
Rel. freq.	0	0	$\frac{1}{42}$	0	$\frac{3}{42}$	$\frac{6}{42}$	$\frac{8}{42}$	$\frac{10}{42}$	$\frac{7}{42}$	$\frac{5}{42}$	$\frac{2}{42}$

$$P(x=8) = \frac{7}{42} \approx 0.167$$

$$P(x \geq 9) = \frac{5}{42} + \frac{2}{42} \approx 0.167$$

Example Explain why the following are not valid PDF's.

(i)

$x$	0	1	2	3	4	5
$p(x)$	0	0.1	0.3	0.6	0.2	0.1

(ii)

$x$	a	b	c	d	e	f
$p(x)$	0	0.1	-0.3	0.6	0.4	0.2

In (i)  $\sum p(x) = 1.3$ . This is not 1.

In (ii)  $p(c) = -0.3$ . This is not  $\geq 0$ .