# §4.3 Expected Values of a Discrete Random Variable

<u>Definition</u> The <u>mean</u>, or <u>expected value</u>, of a discrete random variable $x$ is

$$\mu = E(x) = \sum x\, p(x)$$

<u>Example</u> A multiple choice exam has questions with five possible choices. On each problem choosing the correct answer gains the student 7 points, while choosing an incorrect answer losses the student 4 points. leaving a question blank is worth 0 points. What is the Probability Distribution on each question that is answered randomly (guessing) and what is the expected value for the "score" of a randomly answered question?

Here the random variable, $X$, is the score the student receives on a randomly answered question. That is $x$ takes on values in $\{7, -4\}$. The distribution function is

| $x$ | 7 | -4 |
|---|---|---|
| $p(x)$ | $\frac{1}{5}$ | $\frac{4}{5}$ |

and the expected value is

$$E(x) = (7)\left(\tfrac{1}{5}\right) + (-4)\left(\tfrac{4}{5}\right) = -\tfrac{9}{5} = -1.8$$

Note: On this type of exam it is better to leave a question blank, than it is to randomly guess.

**Definition** The variance of a random variable $X$ is

$$\sigma^2 = E[(x-\mu)^2] = \sum (x-\mu)^2 p(x) = \left(\sum x^2 p(x)\right) - \mu^2$$

The standard deviation of a discrete random variable is

$$\sigma = \sqrt{\sigma^2}$$

**Example** Find the mean and standard deviation of the following

| $x$ | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|
| $P(x)$ | .2 | .3 | .25 | .15 | .1 |

$$\mu = (10)(.2) + (15)(.3) + (20)(.25) + (25)(.15) + (30)(.1) = 18.25$$

$$\sigma^2 = \left((10)^2(.2) + (15)^2(.3) + (20)^2(.25) + (25)^2(.15) + (30)^2(.1)\right) - 18.25^2$$

$$= 38.1875$$

$$\sigma = 6.1796$$

Probability Rule for a Discrete Random Variable:
(see page 198 in the book)

Empirical Rule: for symmetric, mound shaped data we have

$$P(\mu - \sigma < x < \mu + \sigma) \approx .68$$

$$P(\mu - 2\sigma < x < \mu + 2\sigma) \approx .95$$

$$P(\mu - 3\sigma < x < \mu + 3\sigma) \approx 1.00$$

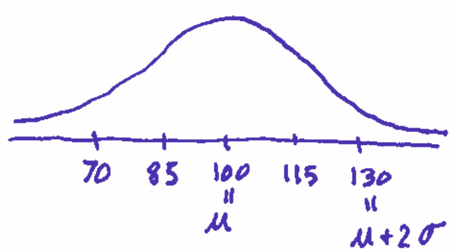
Chebyshev's Rule: Any probability distribution

$$P(\mu - \sigma < x < \mu + \sigma) \geq 0$$

$$P(\mu - 2\sigma < x < \mu + 2\sigma) \geq 3/4$$

$$P(\mu - 3\sigma < x < \mu + 3\sigma) \geq 8/9$$

Example A random variable $X$ has a PDF that is symmetric and mound shaped. The mean is 100 and the standard deviation is 15. What is the probability that a randomly selected element has a value above 130?




By the Empirical Rule

$$.95 = P(\mu-2\sigma < x < \mu+2\sigma)$$
$$= P(70 < x < 130)$$

so $P(x<70 \text{ or } x>130) \approx 1-.95 = .05$

Since the PDF is symmetric,

$$P(x>130) \approx \tfrac{1}{2}(.05) = .025 = 2.5\%$$