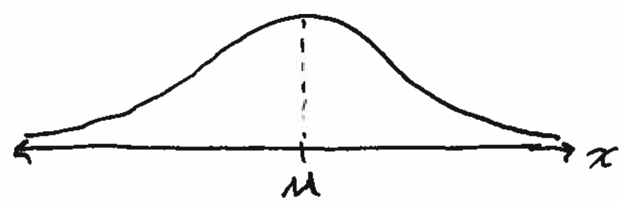
§5.3 The Normal Distribution

One of the most commonly observed continuous random variables has a bell-shaped probability distribution (or bell curve). This type of distribution is called a normal distribution.



The Empirical Rule is based on the normal probability distribution. The probability density function for a normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where

μ = Mean of the normal randon variable x

σ = standard deviation of x

π and e are mathematical constants.

We use z-scores to convert normal distributions into a standard normal distribution.

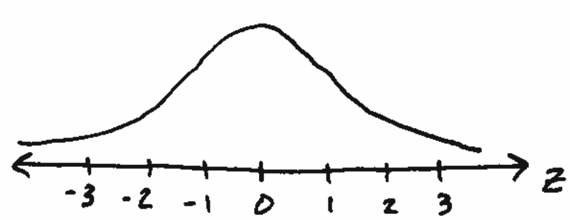


Table II in Appendix B gives $P(0 < z < a)$ for a values out to two decimal digits.

<u>Example</u> Scores on an exam are symmetric and mound shaped. Assuming that the exam scores are normally distributed with mean of 70.7 and standard deviation of 9.9, find the probability that a randomly sellected ~~exam~~ student scored in the following intervals

(a) $70.7 < x < 80$

(b) $60 < x < 80$

(c) $x \geqslant 90$

First calculate some z-scores:

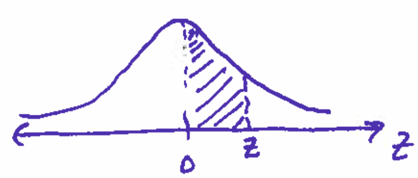
$x = 70.7 \Rightarrow z = 0$ b/c 70.7 was the mean.

$x = 80 \Rightarrow z = \dfrac{80 - 70.7}{9.9} = 0.939393\ldots \approx .94$

$x = 60 \Rightarrow z = \dfrac{60 - 70.7}{9.9} = -1.080808\ldots \approx -1.08$

$x = 90 \Rightarrow z = \dfrac{90 - 70.7}{9.9} = 1.949494\ldots \approx 1.95$

The Table on page 816 is for the graph

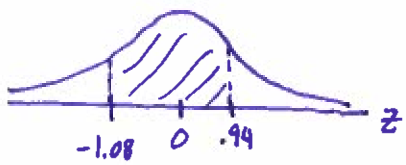


So (a) $P(0 < z < 0.94) = .3264$

the calculator gives .3262357562

The difference is because we used a z-score of .94 instead of .939393...

(b)



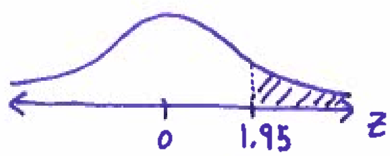
Because of the symmetry,

$P(-1.08 < z < .94) = P(0 < z < 1.08) + P(0 < z < .94)$

$\overset{\text{Table}}{=} .3599 + .3264 = .6863$

From the calculator: .6863444756

(c)



So to use the table,

$P(z \geq 1.95) = .5 - P(0 < z < 1.95)$

$= .5 - .4744 = .0256$

 $= .0256181029$

Now let's turn it arround and find z-scores based on probabilities.

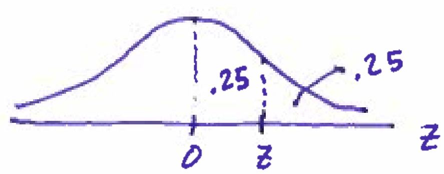
Example An automobile manufacturer designs a vehicle with an average highway gas mileage of 30 mpg and with a stadard deviation of 2.7 mpg. Find the values of α for this type of vehicle given that

(a) $P(30 < x < \alpha) = .25$

(b) $P(30 - \alpha < x < 30 + \alpha) = .8$

(c) $P(x < \alpha) = .05$

For part (a):



In the Table $P(0 < z < .67) = .2486$ and $P(0 < z < .68) = .2517$

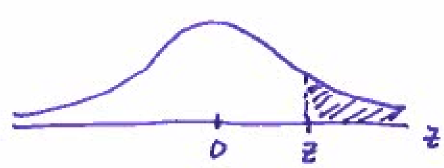
So $z \approx .675$. Recall that

$$z = \frac{x - \mu}{\sigma} \iff x = z\sigma + \mu$$

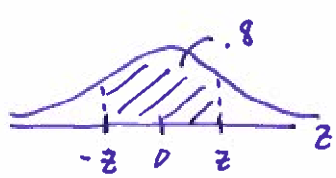
So $\alpha = (.675)(2.7) + 30 = 31.8225$

invNorm  $= 31.82112232$

Note: when using the tails on invNorm the graph is



(b)



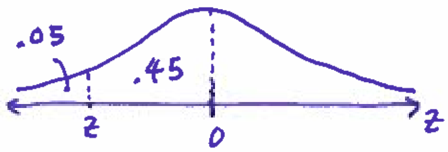
In the Table $P(0 < z < 1.28) = .3997$

So $\alpha = (1.28)(2.7) + 30 = 33.456$

or $P(26.5 < x < 33.5) \approx .8$

 $\Rightarrow$ $P(26.54 < x < 33.46) \approx .8$

(c)



Use symmetry. In the table $P(0 < z < 1.645) = .45$

So ~~$x = -(1.645)(2.7)+30) =$~~ $z = -1.645$

and $x = (-1.645)(2.7) + 30 = 25.5585$

thus $P(x < 25.6) \approx .05$

 $P(x < 25.5588952 1) = .05$

Final Thoughts:

- Draw the picture!
- Use the Empirical Rule as a guide!
- Use symmetry.
- Know how to convert back and forth between raw scores and z-scores.