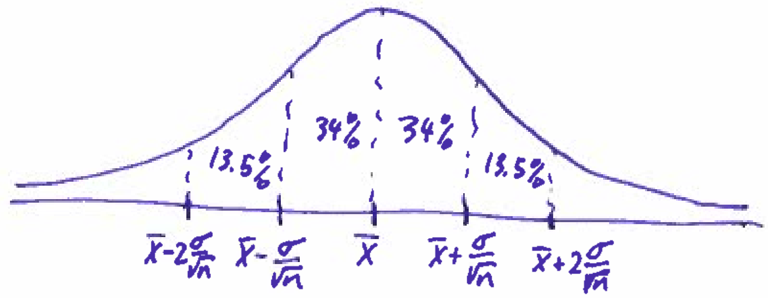
7.2 Examples

(i) Example suppose $\bar{X} = 5.7$, $n = 500$ and we know $\sigma = 1.3$. Find the 95% confidence interval for μ (assuming the population is approximately normally distributed).

(ii) Example Assuming the population is normally distributed and a sample of 1000 elements has a mean of 23.5 and standard deviation of 3.1, find a confidence interval with $\alpha = .09$. What is the confidence level?

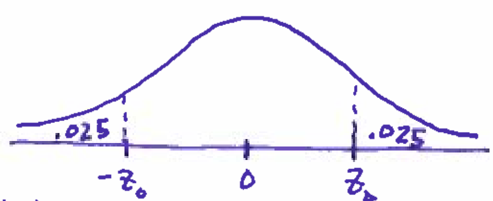
(iii) Example A survey of 100 ~~10,000~~ american households finds that the variance of weekly grocery expenses is 230 square dollars. If an interval of average household spending of this type is reported to be (67, 73) dollars, then what was the α that was used?

In these example problems, remember that the picture looks like



So on

(i) 95% confidence interval meas $\alpha = .05$. Then $\alpha/2 = .025$.
the picture using z-scores is



In the calculator:

$$z_0 = \text{invNorm}(.025, 0, 1, \text{RIGHT}) = 1.96$$

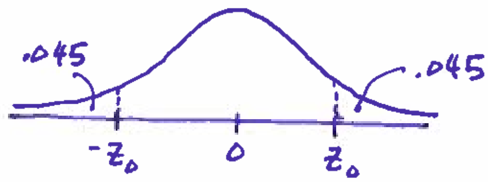
then

$$\bar{X} + z_0\left(\frac{\sigma}{\sqrt{n}}\right) = 5.7 + (1.96)\left(\frac{1.3}{\sqrt{500}}\right) = 5.81395$$

$$\bar{X} - z_0\left(\frac{\sigma}{\sqrt{n}}\right) = 5.7 - (1.96)\left(\frac{1.3}{\sqrt{500}}\right) = 5.58605$$

So the 95% confidence interval for μ is approximately $(5.59, 5.81)$ or $5.59 < \mu < 5.81$.

(ii) $\bar{X} = 23.5$, $s = 3.1$, $n = 1000$, $\alpha = .09$ (so 91% confidence interval and $\alpha/2 = .045$)



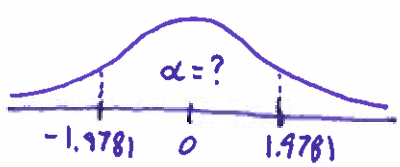
$z_0 = \text{invNorm}(.045, 0, 1, \text{RIGHT}) = 1.695$

Then
$$\bar{X} + z_0\left(\frac{s}{\sqrt{n}}\right) = 23.5 + (1.695)\left(\frac{3.1}{\sqrt{1000}}\right) = 23.66616$$
$$\bar{X} - z_0\left(\frac{s}{\sqrt{n}}\right) = 23.5 - (1.695)\left(\frac{3.1}{\sqrt{1000}}\right) = 23.333838$$

So the 91% confidence interval for μ is $(23.3, 23.7)$ or $23.3 < \mu < 23.7$.

(iii) $n = \overset{100}{\cancel{10{,}000}}$, $s = \sqrt{230} = 15.16575$, confidence interval is $(67, 73) \Rightarrow \bar{X} = \frac{67+73}{2} = 70$. Thus

$$\bar{X} + z_0\left(\frac{s}{\sqrt{n}}\right) = 73 \Rightarrow 70 + z_0\left(\frac{15.16575}{\sqrt{\underset{100}{\cancel{10{,}000}}}}\right) = \cancel{70}\ 73$$
$$\Rightarrow \quad z_0 = (73 - 70)\left(\frac{\overset{\sqrt{100}}{\cancel{\sqrt{10{,}000}}}}{15.16575}\right) = 1.9781$$



$$\alpha = \text{normalcdf}(-1.9781, 1.9781, 0, 1) = 0.95208$$

So the α used is about .952.

Note: α is called the type I error in estimating μ.