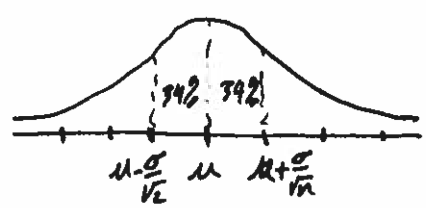
§7.2 Confidence Interval for a Population Mean: Normal (z) Stat.

the basic idea is to construct an interval where $P(a < \mu < b) = 1-\alpha$. The way we do this relies on something called The Central Limit Theorem.

This theorem states that for large samples, the test statistic $\overline{X}$ is approximately normal, with mean μ and standard deviation of $\frac{\sigma}{\sqrt{n}}$

That is, the distribution for $\overline{X}$, where the populatin is sampled in groups of n, is

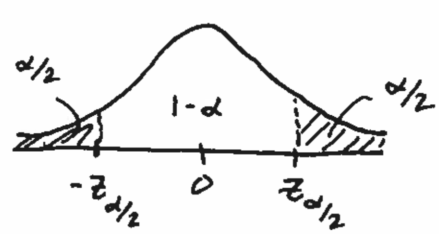


recall that $z = \frac{x-\mu}{\sigma} \iff x = \mu + z\sigma$.

So if we want $P(a<\mu<b) = 1-\alpha$, then

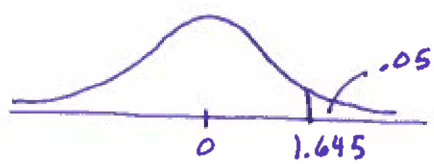
$$a = \overline{X} - z_{\alpha/2}\sigma_{\overline{x}} \quad \text{and}$$
$$b = \overline{X} + z_{\alpha/2}\sigma_{\overline{x}}$$

Where $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$ and $z_{\alpha/2}$ is choosen so that



Example Construct the 90% confidence interval for the average life of a light bulb, where 100 bulbs are tested and this sample had a mean life of 250 hours with a standard deviation of 10.75 hours.

Here $\alpha = 10\%$, so



So $$\bar{X} - z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) = 250 - (1.645)\left(\frac{10.75}{\sqrt{100}}\right)$$
$$= 248.23$$

and $$\bar{X} + z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) = 250 + (1.645)\left(\frac{10.75}{\sqrt{100}}\right)$$
$$= 251.77$$

the confidence interval for the population mean is ~~251.77,~~ (248.23, 251.77) or $248.23 < \mu < 251.77$ at the 90% confidence level.

Confidence level	α	$\alpha/2$	$z_{\alpha/2}$
90%	.1	.05	1.645
95%	.05	.025	1.960
98%	.02	.01	2.326
99%	.01	.005	2.575