§7.3 Confidence Intervals for Population Mean: Student's t-Statistic.

Requirements:

(1) the population is normal

(2) σ is almost always unknown, so use s instead.

The t-statistic generally has 2 unknows: $\bar{x}$ and s (or μ and σ). The z-statistic generally has only one unknown: $\bar{x}$ (or μ). In the z-statistic we can use s for σ because n is large.

Definition for the t-statistic, the degrees of freedom is $(n-1)$.

On page 817, table III, gives t_α-scores for $\alpha = .1, .05, .025, .005, .001$ and $.0005$ with degrees of freedom $1, 2, 3, \ldots, 30$ (also $40, 60, 120$, and ∞).

For "small" samples, the $100(1-\alpha)$ confidence interval for μ is

$$\bar{x} \pm (t_{\alpha/2})\left(\frac{s}{\sqrt{n}}\right) \quad \text{when } \sigma \text{ is unknown}$$

$$\bar{x} \pm (z_{\alpha/2})\left(\frac{\sigma}{\sqrt{n}}\right) \quad \text{when } \sigma \text{ is known}$$

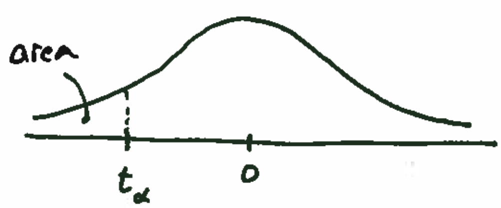
Example A sample of size 20 is taken. $\bar{x} = 10.2$, $s = 1.6$. We do not know σ. Find the 95% confidence interval for μ.

In this case $\alpha = .05$, so $\frac{\alpha}{2} = .025$. $n = 20$ means we have 19 degrees of freedom. On the chart $t_{.025} =$ 2.093 (when df = 19). Thus

$$\bar{x} \pm (t_{.025})\left(\frac{s}{\sqrt{n}}\right) = 10.2 \pm (2.093)\left(\frac{1.6}{\sqrt{20}}\right)$$

$\Rightarrow$ $(9.45, 10.95)$ or $9.45 < \mu < 10.95$.

Note: invT (area, df) gives



So in the above invT(.025,19) = −2.093

Example A sample of size 15 yields a standard deviation of 2.2. A confidence interval of (20, 22) is generated for μ. What α was used?

$n = 15 \Rightarrow df = 14$. $s = 2.2$ and $\bar{x} = \frac{20+22}{2} = 21$.
That means

$$\bar{x} + (t_{\alpha/2})\left(\frac{s}{\sqrt{n}}\right) = 22 \Rightarrow$$

$$21 + (t_{\alpha/2})\left(\frac{2.2}{\sqrt{15}}\right) = 22 \Rightarrow$$

$$t_{\alpha/2} = (22-21)\left(\frac{\sqrt{15}}{2.2}\right) = 1.7604$$

On page 817 for df = 14, $t_{.05} = 1.761$. Thus $\alpha/2 = .05$ and $\alpha = .1$. Note this is a 90% confidence interval for μ.