

## §8.1 The Elements of a Test of Hypothesis

In this chapter we want to determine, with some degree of certainty ~~whether~~ if a population parameter meets some predetermined value.

### Examples

- (i) A new fertilizer makes lawns "greener".
- (ii) A drug to treat cancer has fewer side-effects.
- (iii) A type of light bulb has a certain average lifespan.

When investigating claims like these, we always form two competing statements or hypothesis.

Definitions • A statistical hypothesis is a statement about the numerical value of a population parameter.

- The null hypothesis, denoted  $H_0$ , represents the hypothesis that will be assumed to be true unless the data provides convincing evidence that it is false. This usually represents the "status quo" for the parameter.

- The alternative (research) hypothesis, denoted  $H_a$ , represents the hypothesis that will be "accepted" only if the data provides convincing evidence that it is likely true. This usually represents the result the researcher is trying to support.

### Examples

- (i)  $H_0: \mu \leq a$ , for some constant  $a$   
 $H_a: \mu > a$ .
- (ii)  $H_0: \mu \geq a$ , for some constant  $a$   
 $H_a: \mu < a$ .
- (iii)  $H_0: \mu = a$ , for some constant  $a$   
 $H_a: \mu \neq a$ .

In (i)  $H_a$  could be "A new fertilizer makes lawns greener than some fixed value of color".

In (ii)  $H_a$  could be "A drug to treat cancer has fewer than a fixed number of side effects".

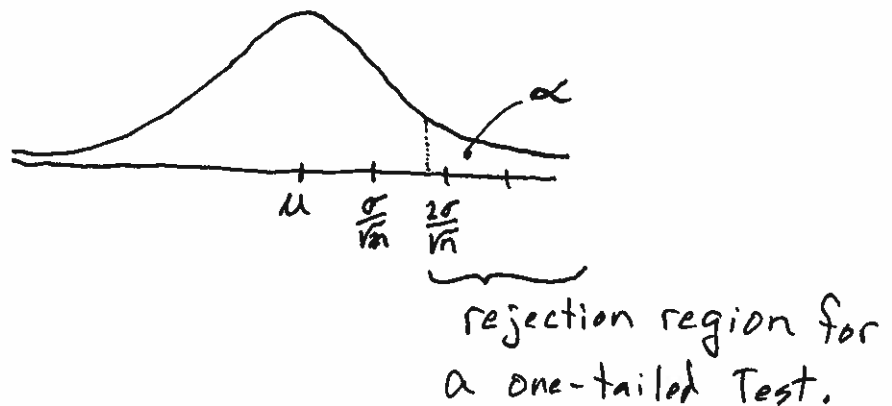
In (iii)  $H_a$  could be "A light bulb has a different average life span, than other types of bulbs".

Definition The test statistic is a sample statistic, computed from information provided in the sample, that researchers use to decide between the null and alternative hypothesis.

The results of the hypothesis test should always be either: (1) The data supports rejecting the null hypothesis, or (2) The data fails to support rejecting the null hypothesis. In general, we never "accept" a hypothesis as being true!!!

Definition A Type I error occurs if the researcher rejects the null hypothesis when, in fact,  $H_0$  is true. The probability of committing a Type I error is denoted as  $\alpha$ .

As before the variable  $\bar{X}$  should be normally distributed with standard deviation  $\sigma/\sqrt{n}$ .



Definition the rejection region of a statistical test is the set of possible values of the test statistic for which the researcher will reject  $H_0$ .

Definition A Type II error occurs if a researcher fails to ~~accept~~ reject the null hypothesis, when in fact,  $H_0$  is false. The probability of committing a type II error is denoted as  $\beta$ .

$\beta$  is often difficult to determine!

Please look at The Elements of a test of Hypothesis on page 374 of the book.