8.2 Formulating Hypotheses and Setting Up the Rejection Region

1. Select the H_a as that which the sampling experiment is intended to establish. H_a will take one of three forms:
   (a) One-tailed, upper-tailed ($H_a: \mu > a$)
   (b) One-tailed, lower-tailed ($H_a: \mu < a$)
   (c) Two-tailed ($H_a: \mu \neq a$)
2. Select the H_0 as the status qoa. H_0 is the compliment of H_a.
   (a) $H_0: \mu \leq a$ or $\mu = a$
   (b) $H_0: \mu \geq a$ or $\mu = a$
   (c) $H_0: \mu = a$

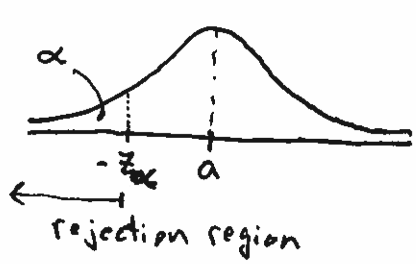
The Test statistic is a z-score. For averages,

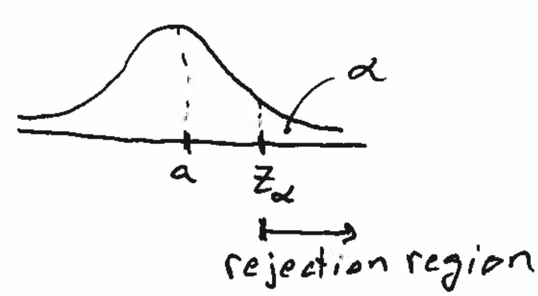
$$z = \frac{\bar{x} - a}{\sigma_{\bar{x}}} = \frac{\bar{x} - a}{\sigma / \sqrt{n}}$$

Where $\bar{x}$ is the mean value of a sample of n elements and $\sigma \approx s$ if σ is not known.

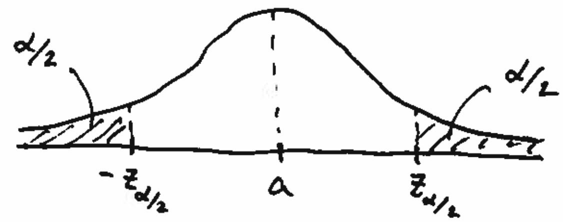
Rejection Region is based on α and the type of test, i.e. One or Two tailed.

One-tailed:





Two-tailed



▨ - rejection region.

If the calculated z lies in the rejection region, the reject H_0. Remember that this carries with it the chance of committing a Type I error.

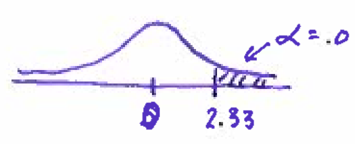
If the calculated z is not in the rejection region, then fail to reject H_0. Remember that accepting H_0 could result in committing a Type II error.

Example A device that we will call a "Greenometer" measures the green in lawns with 0 being no green to 10 being the most amount of green. For untreated lawns $\mu = 5$ and $\sigma = 1.5$. A company claims that their fertilizer "makes lawns more green". A researcher samples 100 lawns that where treated with the fertilizer and find the $\bar{x} = 5.31$. If $\alpha = .01$, what can we conclude about the companies claims?

$H_0: \mu = 5$ for treated lawns.

$H_a: \mu > 5$ for treated lawns.

At $\alpha = .01$, $z_\alpha = 2.33$



Test statistic $z = \dfrac{\bar{x} - 5}{\sigma/\sqrt{n}} = \dfrac{5.31 - 5}{1.5/\sqrt{100}} = 2.0667$

z is not in the rejection region, so we fail to reject the null hypothesis. That is, the data does not support the companies claims at $\alpha = .01$.

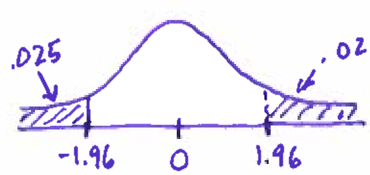
Note: $z_{.05} = 1.645$, so if the researchers had started with $\alpha = .05$, then we reject the null hypothesis. However, it is considered "bad form" to change α after taking your sample.

<u>Example</u> An educator claims that sleeping 8 or more hours per night (instead of studying or playing Call of Duty) has an effect on a students G.P.A. If the G.P.A of all students has a mean of 2.75 and standard deviation of 0.6. A researcher decides to use $\alpha = .05$ and gets 150 randomly selected students to volunteer to sleep at least 8 hours each night of a semester and finds that the average G.P.A. for these students was 2.85 for that semester. What can the researcher say about the claim?

$H_0: \mu = 2.75$ for student sleeping 8 hours per night

$H_a: \mu \neq 2.75$

At $\alpha = .05$, $z_{\alpha/2} = 1.96$



Test statistic: $z = \dfrac{\bar{x} - 2.75}{\sigma/\sqrt{n}} = \dfrac{2.85 - 2.75}{0.6/\sqrt{150}} = 2.0412.$

z is in the rejection region, so we reject H_0 and say the data supports the researchers claim when $\alpha = .05$.