

§ 8.3 Observed Significance Levels: p-values

As was seen in the previous section, the choice of α can make a difference in the test either rejecting or failing to reject H_0 . In practice it is often better to report the observed significance level for the test and leave the conclusion to the reader.

Def The observed significance level, or p-value, for a specific statistical test is the probability (assuming H_0 is true) of observing a value of the test statistic that is at least as contradictory to the null hypothesis, and supportive of the alternative hypothesis, as the actual one computed from the sample data.

Example In the previous section notes, the example on sleeping students had ~~the~~ $H_0: \mu = 2.75$, $H_a: \mu \neq 2.75$.
 $\sigma = .6$
 the sample had $\bar{X} = 2.85$. Find the p-value.

Here we will use the stat → tests → z-test,
Enter in our statistics and calculate to get:

$$\mu \neq 2.75$$

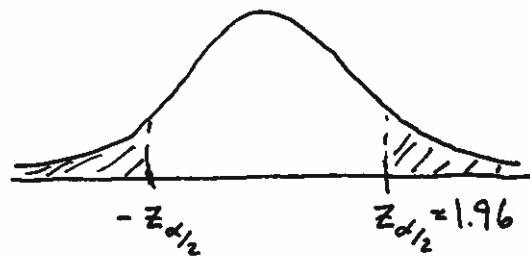
$$Z = 2.0412 \quad \text{which is what we calculated}$$

$$P = 0.0412 \quad \leftarrow \text{the value ask for in the problem}$$

$$\left. \begin{array}{l} \bar{X} = 2.85 \\ n = 150 \end{array} \right\} \text{ reminding you of what we used.}$$

For what we did in section 8.2, $Z > Z_{\alpha/2}$ or $p < \alpha = .05$,
So we reject the null hypothesis and say the data supports H_a at an observed significance level of $p = 0.0412$.

In the example



$$z = 2.0412 > z_{\alpha/2}$$
$$p = .0412 < \alpha = .05$$

 - rejection region > p

If the test statistic satisfied $z < -z_{\alpha/2}$ we would still reject H_0 . In either case $p < \alpha \Rightarrow$ reject H_0 , $p > \alpha \Rightarrow$ fail to reject H_0 .

Example In section 8.2 we had the "greener lawn" example where $\mu = 5$, $\sigma = 1.5$. $H_0: \mu = 5$, $H_a: \mu > 5$, and the sample statistic was $\bar{x} = 5.31$. Find the p-value. ($n = 100$).

If we enter the data into stat \rightarrow Tests \rightarrow z-test...
we get:

$$\mu > 5$$

$$z = 2.0667 \quad \text{which we calculated}$$

$$p = 0.01938 \quad \leftarrow \text{the value ask for in this problem.}$$

$$\left. \begin{array}{l} \bar{x} = 5.31 \\ n = 100 \end{array} \right\} \text{ remainder of what we entered.}$$

Since the $\alpha = .01$ we used is smaller than p , we failed to reject H_0 in section 8.2. However we can say that the data supports rejecting H_0 at an observed significance of $p = 0.01938$.