

## §8.6 Large-Sample test for Population proportion

Same as before with z-scores as in the section on confidence intervals.

From a sample:

$X$  = number of successes

$n$  = number of observations sampled

$\hat{p} = \frac{X}{n}$ ,  $\hat{q} = 1 - \hat{p}$ ,  $P_0$  = population proportion (also  $p$ )\*

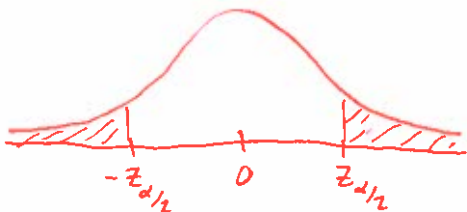
$\alpha$  = significance level

\* not to be confused with  $p$  = observed significance level in hypothesis tests

$$Z_c = \frac{\hat{p} - P_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - P_0}{\sqrt{P_0 Q_0/n}}$$

Confidence Intervals

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

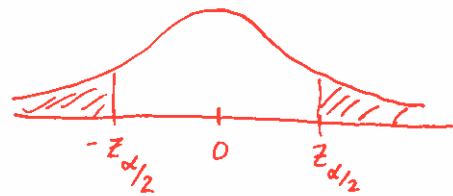


area =  $\alpha$

Hypothesis testing (2-tailed)

$$H_0: P = P_0$$

$$H_a: P \neq P_0$$



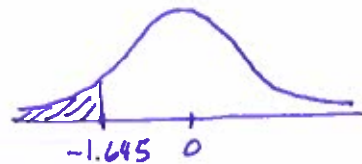
rejection region

If  $Z_c$  is in the rejection region, we reject  $H_0$ . Otherwise, we fail to reject  $H_0$ .

The other (1-tailed) cases are similar.

Example In the year 2000, the proportion of Michigan adults smokers was reported to be 25.5%. The Michigan Department of Public Health conducted a survey in 2002 to assess if the percent of adult smokers had declined. Out of 2400 adults surveyed 595 reported that they smoked. Test the hypothesis that the percent of smokers declined with significance level of .05.

$$Z_{.05} = 1.645 \Rightarrow$$



$$H_0: p = .255$$

$$H_a: p < .255$$

$$x = 595, n = 2400 \Rightarrow \hat{p} = \frac{595}{2400} = .248, \hat{q} = .752$$

Stat  $\rightarrow$  tests  $\rightarrow$  1-PropZtest:

$$p_{prop} < 0.255$$

$$Z_c = -0.796$$

$$P = 0.213 \leftarrow \text{observed significance level}$$

$$\hat{p} = .248$$

$$n = 2400$$

Since  $Z_c$  is not in the rejection region (and the observed significance level is greater than  $\alpha$ ), we fail to reject  $H_0$ . There is not enough evidence to suggest that the proportion of adult smokers has decreased by the year 2002.